Metamaterials and composites: electromagnetic description and unexpected effects

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Hong Kong Chapter — 2 March 2015
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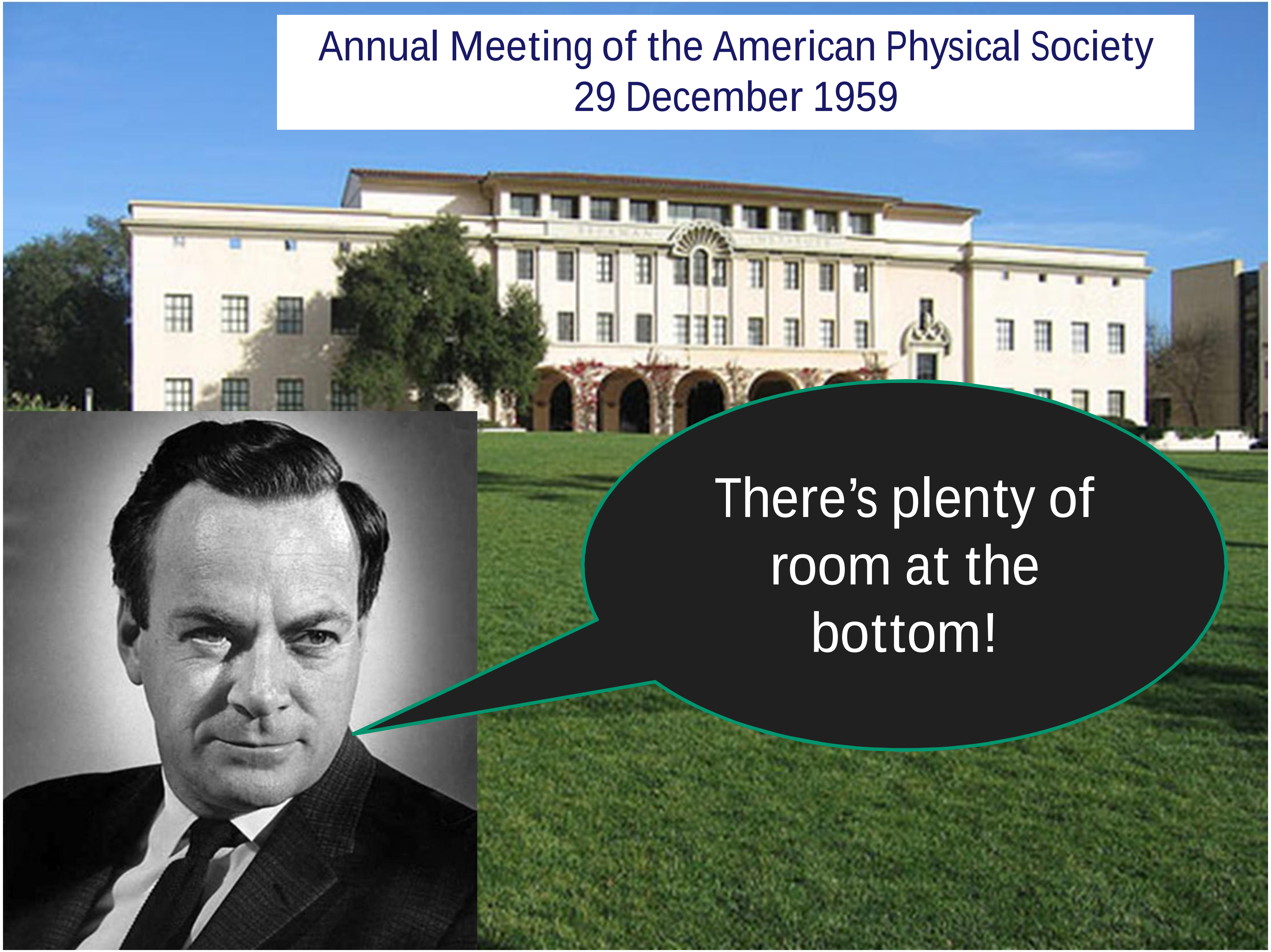
FINLAND
ESPOO


Annual Meeting of the American Physical Society
29 December 1959
There’s plenty of room at the bottom!

Today’s keywords

- scales & levels
- geometry–matter interaction
- emergence/enhancement of losses
- effective description and complex constitutive relations
- mixing rules

METAMATERIALS ?

METAMATERIALS ?

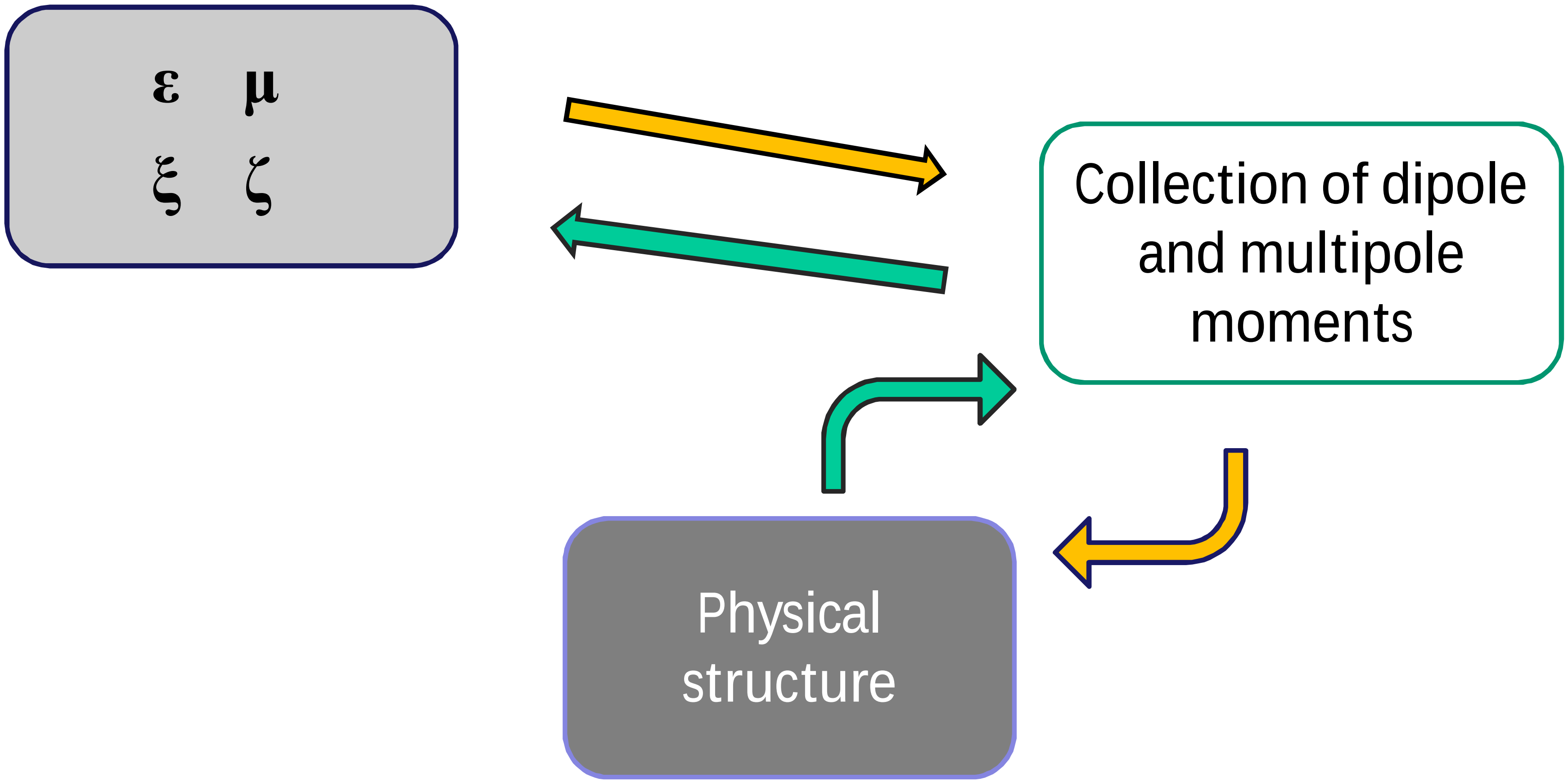
Wikipedia definition (changing...)

2006: In electromagnetism (covering areas like optics and photonics), a meta material (or metamaterial) is an object that gains its (electromagnetic) material properties from its structure rather than inheriting them directly from the materials it is composed of. This term is particularly used when the resulting material has properties not found in naturally-formed substances.

2014: Metamaterials gain their properties not from their composition, but from their exactingly-designed structures. Their precise shape, geometry, size, orientation and arrangement can affect the waves of light or sound in an unconventional manner, creating material properties which are unachievable with conventional materials.

A. Sihvola (2003): Electromagnetic emergence in metamaterials, in *Advances in Electomagnetics of Complex Media and Metamaterials*, (S. Zouhdi *et al*., eds), NATO Science Series, **89**, 1-17. A. Sihvola (2007): Metamaterials in electromagnetics. *Metamaterials*, **1**, 2-11.

Scales in (electromagnetic) material modeling



(meta)material levels and phenomena

- materialization
- realization
- synthetization
- localization
- fabrication

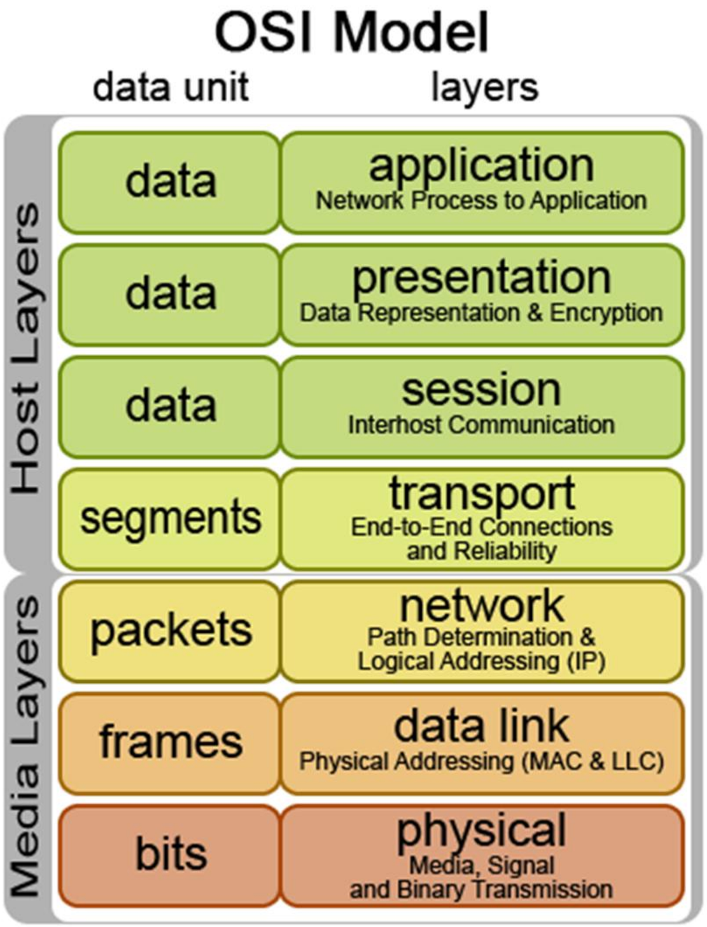
- mathematization
- idealization
- projection
- homogenization
- parametrization
- modularization

- non-unique
- several realizations
- multiple platforms

- wash-out of microsopic structure
- loss of details
- emergence, paradigm change

For example, in EE context...

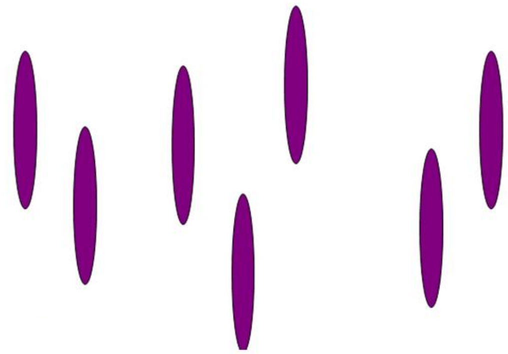
- electromagnetics vs. circuit theory
  - Cellular Neural Networks, non-linear circuits
  - simple rules, complex behavior
- Open Systems Interconnection model for the communication system – abstraction layers



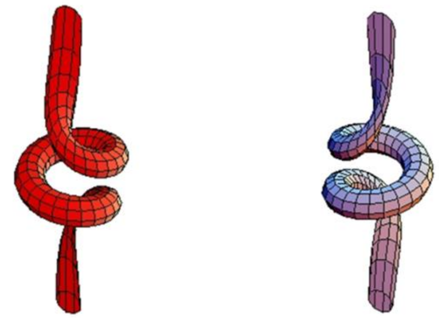
- Emergence: $\boxed{1+1>2}$
- Reductionism: $\boxed{1+1<2}$

- Inheritance of properties
  - children vs. parents
- Multiple platforms (case: computer)
  - the essence of a computer is independent of the technology of the electronic circuits

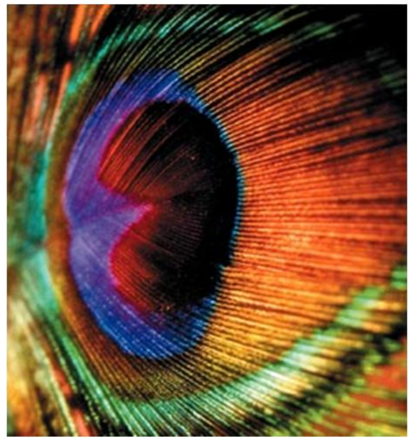
Emergence everywhere!

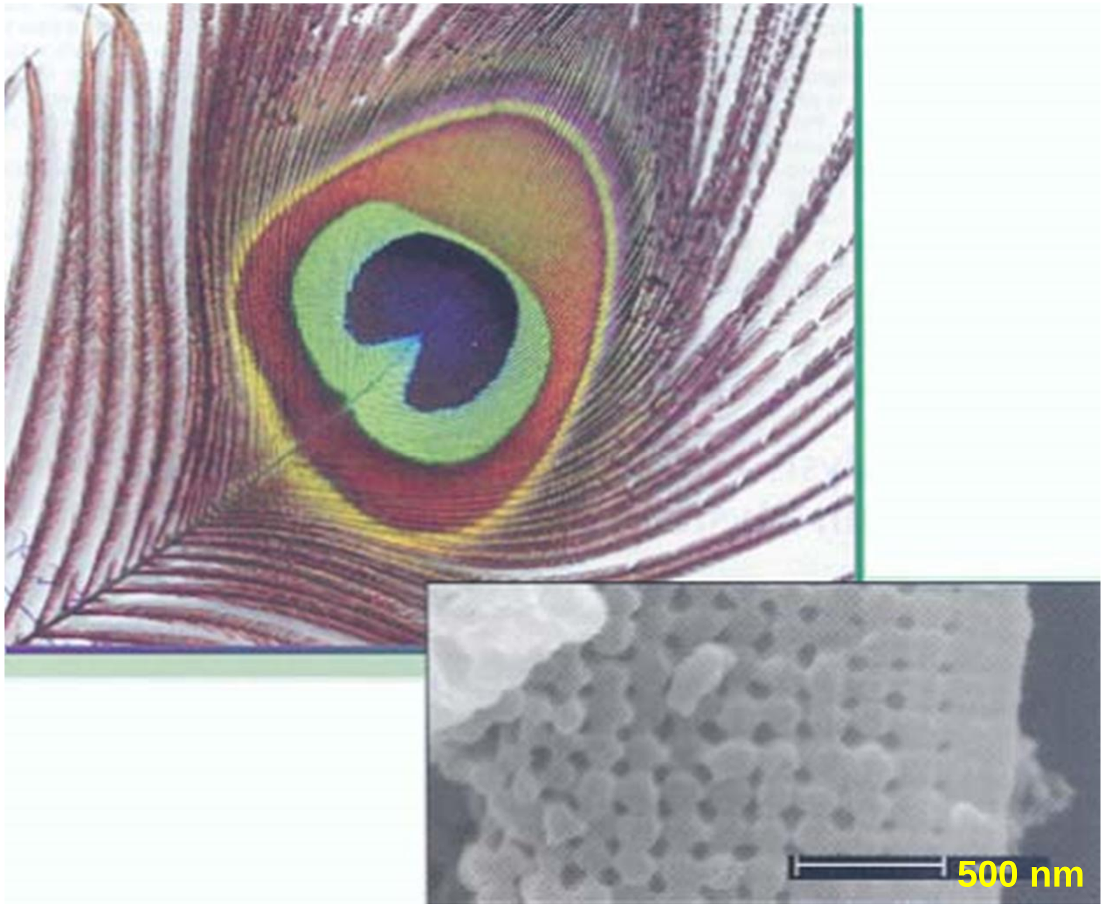


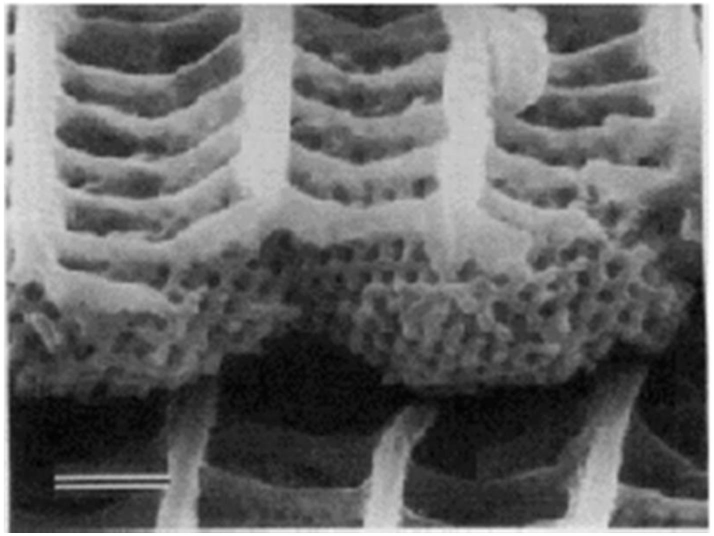
- anisotropy
- chirality and optical activity
- artificial magnetism
- structural colors from photonic band gaps
- ....



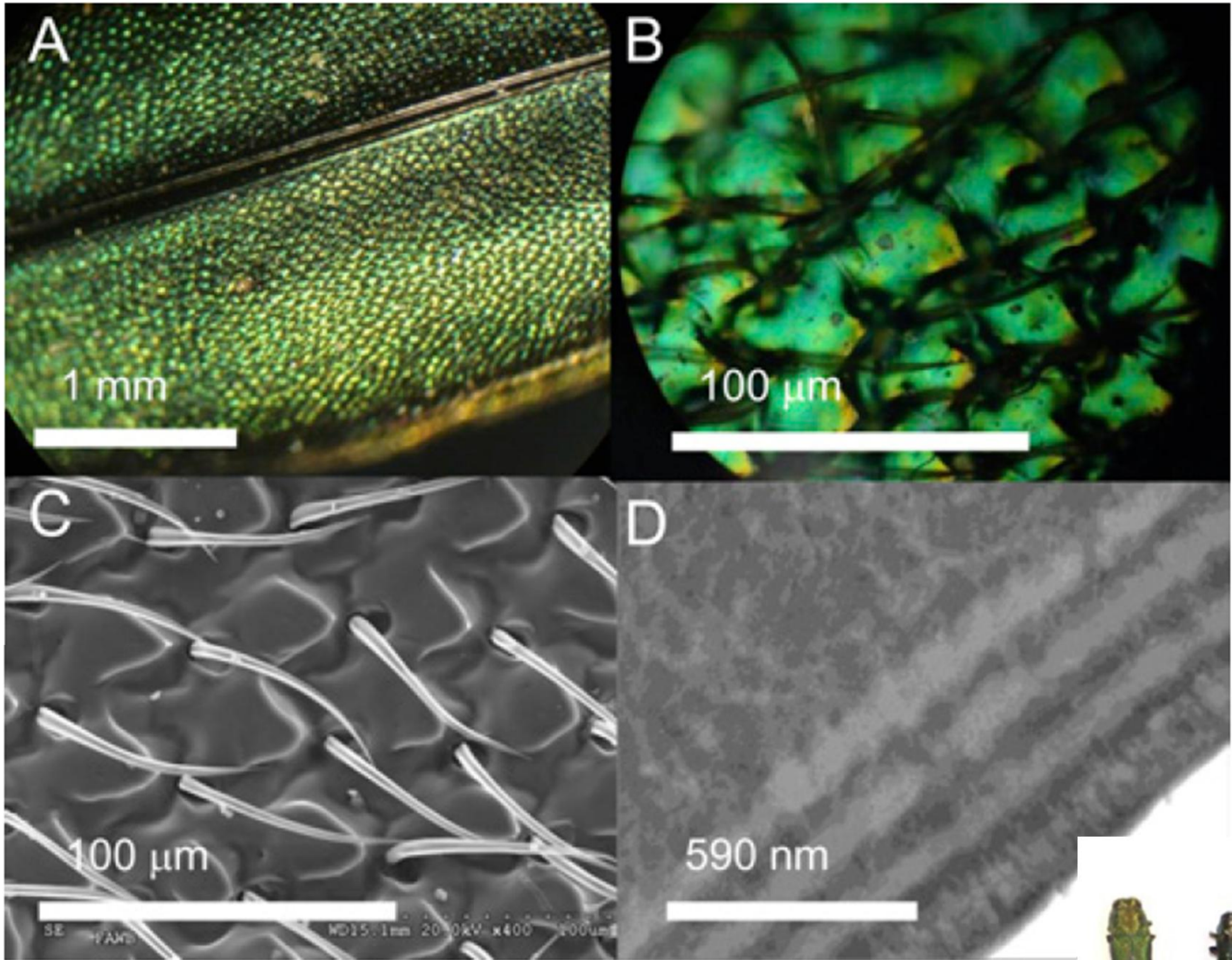
effects determined by the geometry of the small-scale

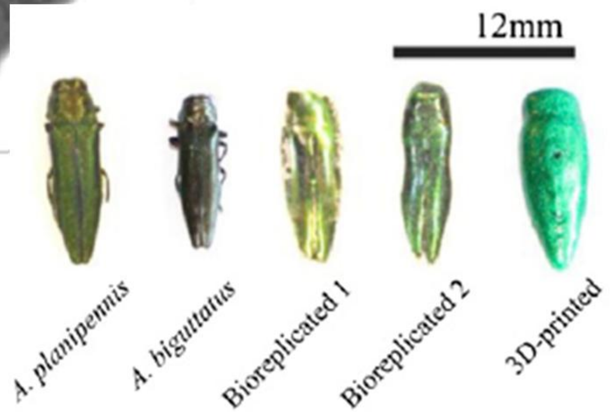






Blau, *Physics Today*
Jan. 2004, p.18-20

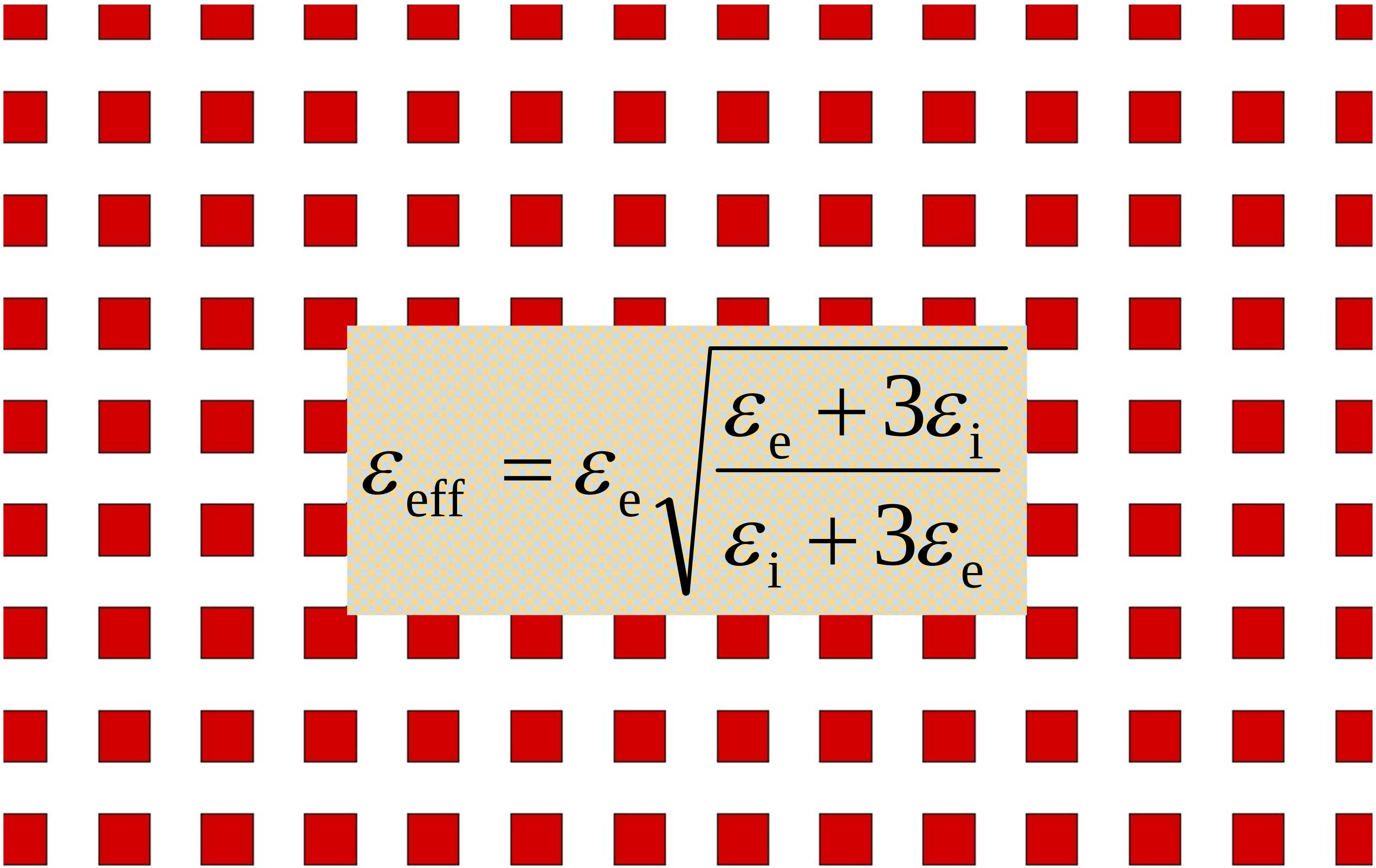




Structural color and surface topography of emerald ash borer beetle (Agrilus planipennis) wings. (Domingue et al., *PNAS*, **111**(39)14106-11, September 30, 2014)

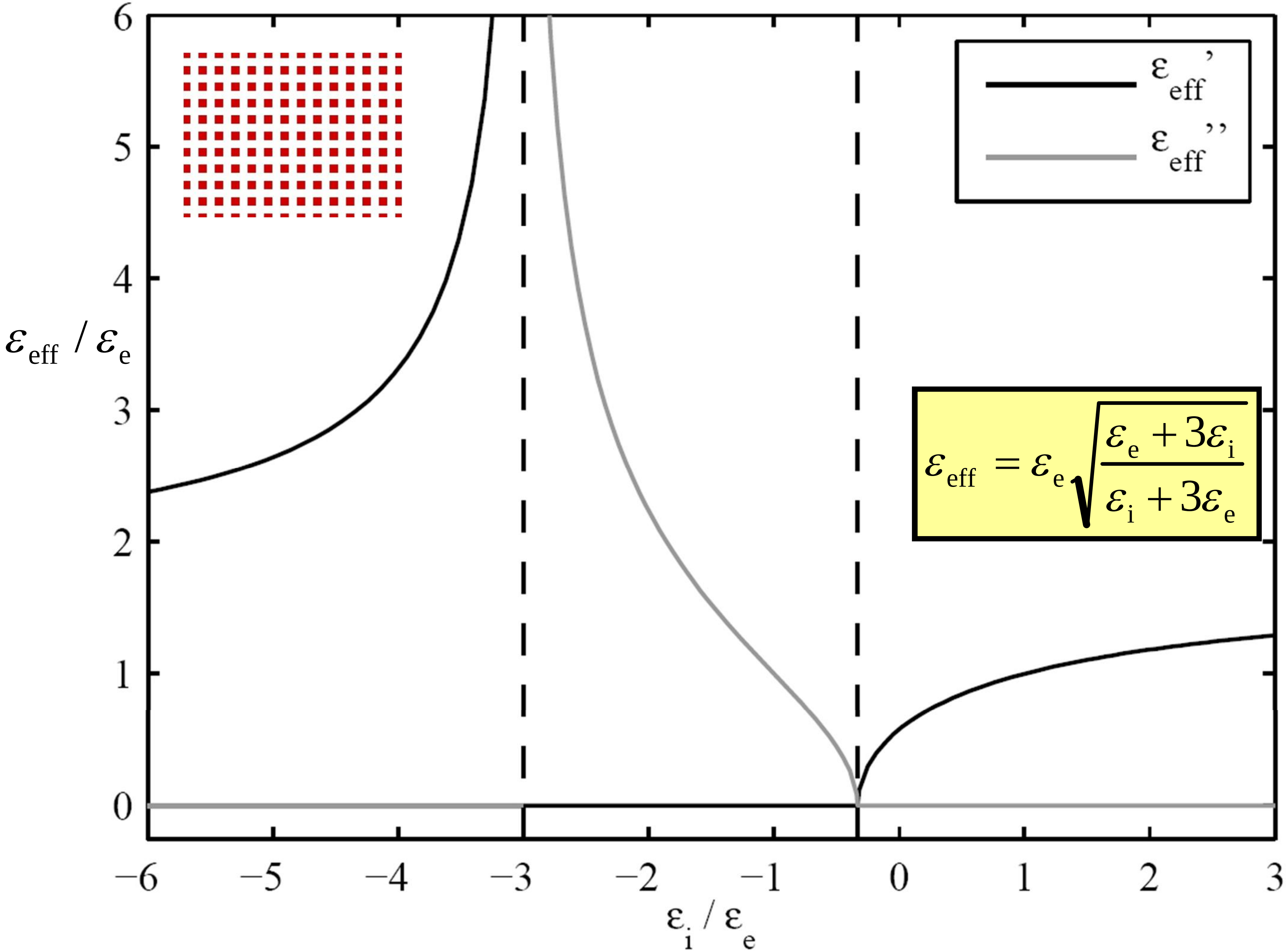
Example

- emergence
  - *losses from lossless building blocks*
  - case of radially anisotropic (RA) sphere



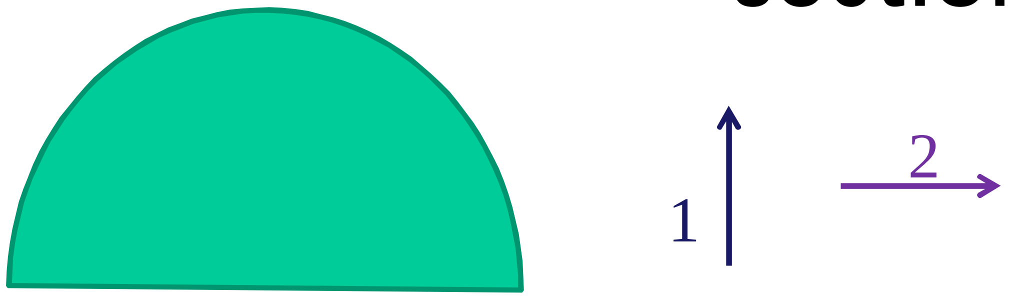
$$\varepsilon_{\mathrm{eff}} = \varepsilon_{\mathrm{e}} \sqrt{\frac{\varepsilon_{\mathrm{e}} + 3\varepsilon_{\mathrm{i}}}{\varepsilon_{\mathrm{i}} + 3\varepsilon_{\mathrm{e}}}}$$

Yu.V. Obnosov. Periodic heterogeneous structures: ”New explicit solutions and effective characteristics of refraction of an imposed field,” *SIAM Journal on Applied Mathematics, 59(4):1267–1287, 1999.*
J. Helsing, R.C. McPhedran, and G.W. Milton: ”Spectral super-resolution in metamaterial composites,” *New J. Physics, 13(11):115005, 2011*.



ε_{eff}'
ε_{eff}''
$\varepsilon_{eff} = \varepsilon_e \sqrt{\dfrac{\varepsilon_e + 3\varepsilon_i}{\varepsilon_i + 3\varepsilon_e}}$
$\varepsilon_{eff} / \varepsilon_e$
$\varepsilon_i / \varepsilon_e$
6
5
4
3
2
1
0
−6
−5
−4
−3
−2
−1
0
1
2
3

Cylinder with semicircular cross section



$$\alpha_1 = \frac{\pi^2(2+3\tau)+12\,\mathrm{Li}_2(\tau^2/2+(\tau/2)\sqrt{\tau^2-4}-1)+12\,\mathrm{Li}_2(\tau^2/2-(\tau/2)\sqrt{\tau^2-4}-1)}{3\pi^2(2+\tau)/4}$$

$$\alpha_2(\varepsilon) = -\alpha_1(1/\varepsilon)$$

α_1, α_2 complex for $-3 < \varepsilon < -1/3$

$$\tau = \frac{\varepsilon - 1}{\varepsilon + 1}$$

M. Pitkonen (2010): *Journal of Electromagnetic Waves and Applications*, **24**, 1267–1277
N. Mohammadi Estakhri and A. Alù (2013): Physics of unbounded, broadband absorption/gain efficiency in plasmonic nanoparticles, *Phys. Rev. B*, **87**, 205418

Losses from lossless: non-dissipative damping?

- chessboard & hemidisk: sharp corners
- continuosly curved shapes?
- RA sphere

Electrostatic response of isotropic (3D) sphere

E

$$\alpha = 3\frac{\varepsilon - 1}{\varepsilon + 2}$$

p

$$\mathbf{p} = \alpha_{\mathrm{abs}}\mathbf{E}$$

$$\alpha = \frac{\alpha_{\mathrm{abs}}}{\varepsilon_0 V}$$

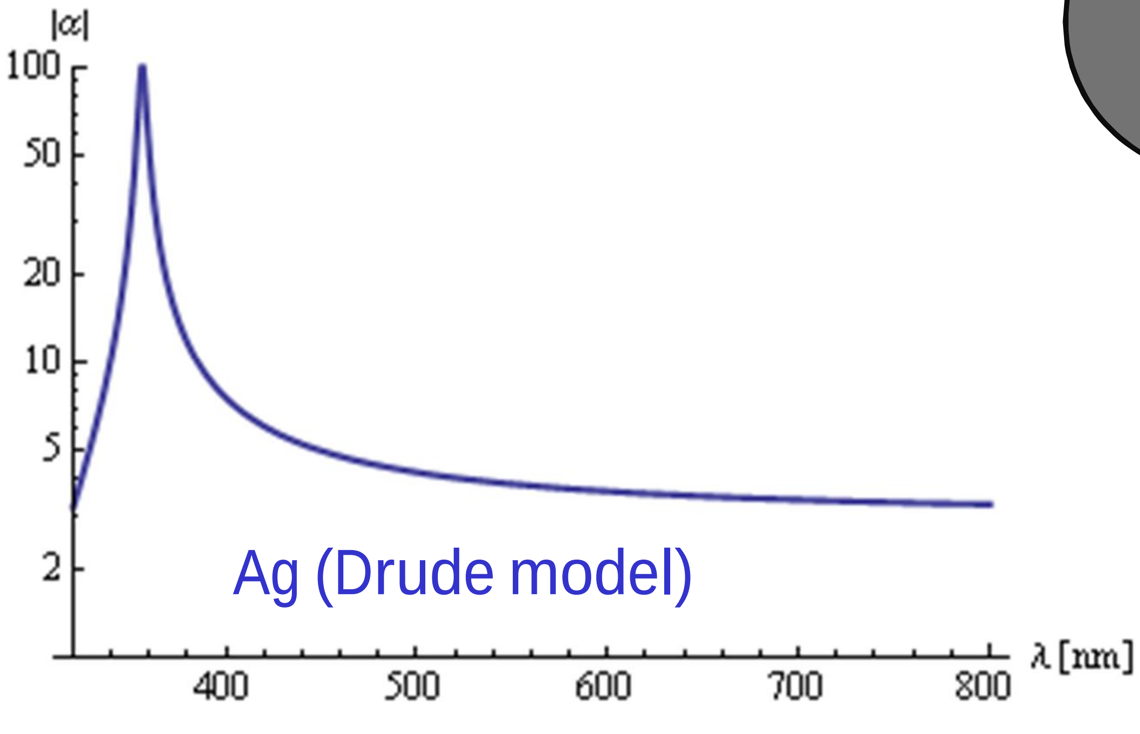
Singularity for
$\varepsilon = -2$

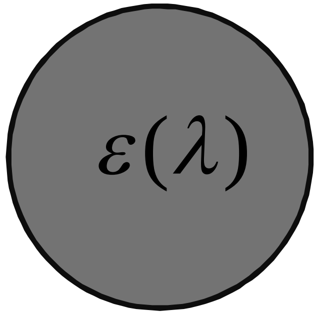
Drude model:

$$\varepsilon(\omega) = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}$$

$$\Rightarrow \omega_{\mathrm{res}} = \frac{\omega_{\mathrm{p}}}{\sqrt{3}}$$

Localized Surface Plasmon, Electrostatic Resonance





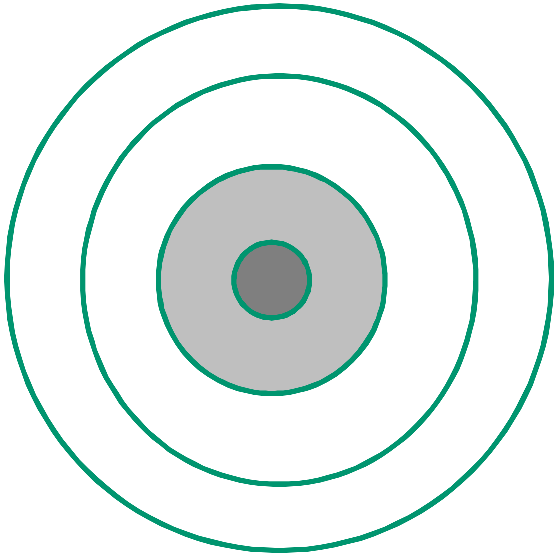
$$\alpha(\lambda) = 3\frac{\varepsilon(\lambda)-1}{\varepsilon(\lambda)+2}$$

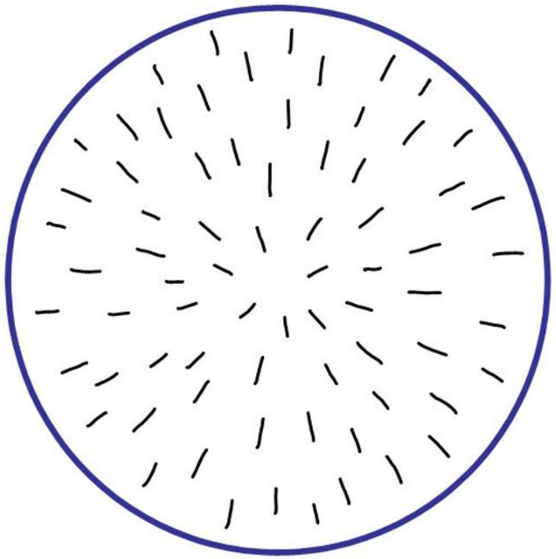
RA sphere: anisotropic

$$\overline{\overline{\varepsilon}} = \varepsilon_r \mathbf{u}_\mathrm{r} \mathbf{u}_\mathrm{r} + \varepsilon_t (\overline{\overline{\mathbf{I}}} - \mathbf{u}_\mathrm{r} \mathbf{u}_\mathrm{r})$$

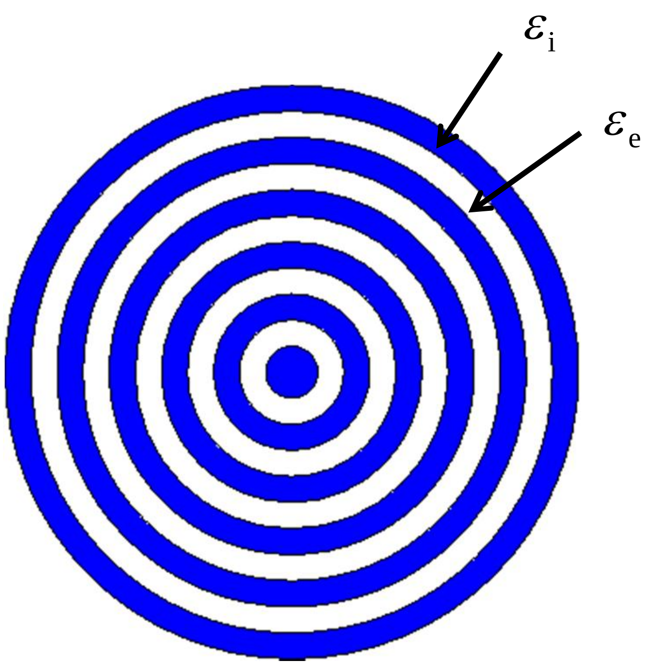
RU/RA *sphere*

– radially uniaxial/anisotropic sphere –





RA sphere from the onion structure



$$\varepsilon_{\mathrm{t}} = p\varepsilon_{\mathrm{i}} + (1 - p)\varepsilon_{\mathrm{e}}$$

$$\varepsilon_{\mathrm{r}} = \frac{1}{p/\varepsilon_{\mathrm{i}} + (1-p)/\varepsilon_{\mathrm{e}}}$$

$$p = \frac{d_{\mathrm{i}}}{d_{\mathrm{i}} + d_{\mathrm{e}}}$$

layer thicknesses d_{i} , d_{e}

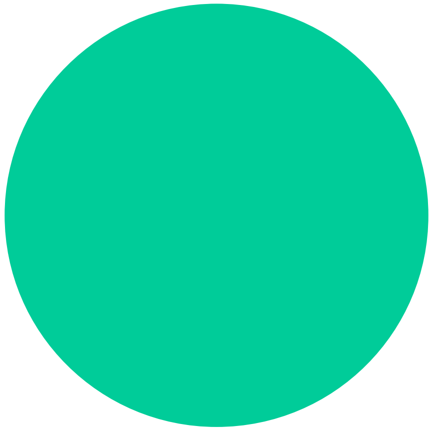
Anisotropy of *RU* sphere

$$\bar{\bar{\varepsilon}} = \varepsilon_r \mathbf{u}_\mathrm{r}\mathbf{u}_\mathrm{r} + \varepsilon_t (\bar{\bar{\mathbf{I}}} - \mathbf{u}_\mathrm{r}\mathbf{u}_\mathrm{r})$$

Dipole response

isotropic: $\bar{\bar{\alpha}} = \alpha \bar{\bar{\mathbf{I}}}$

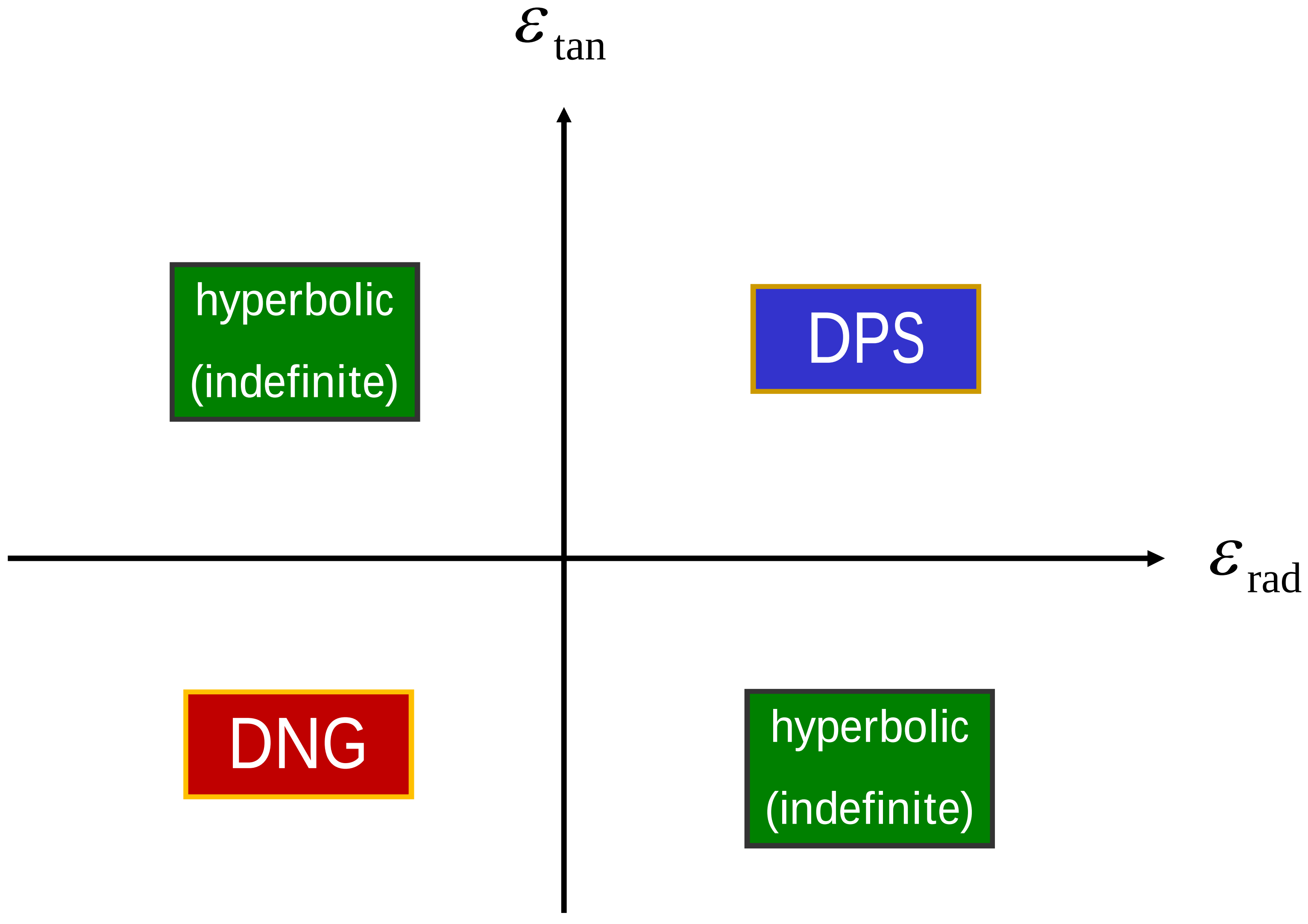
Electrostatic response of isotropic RA sphere

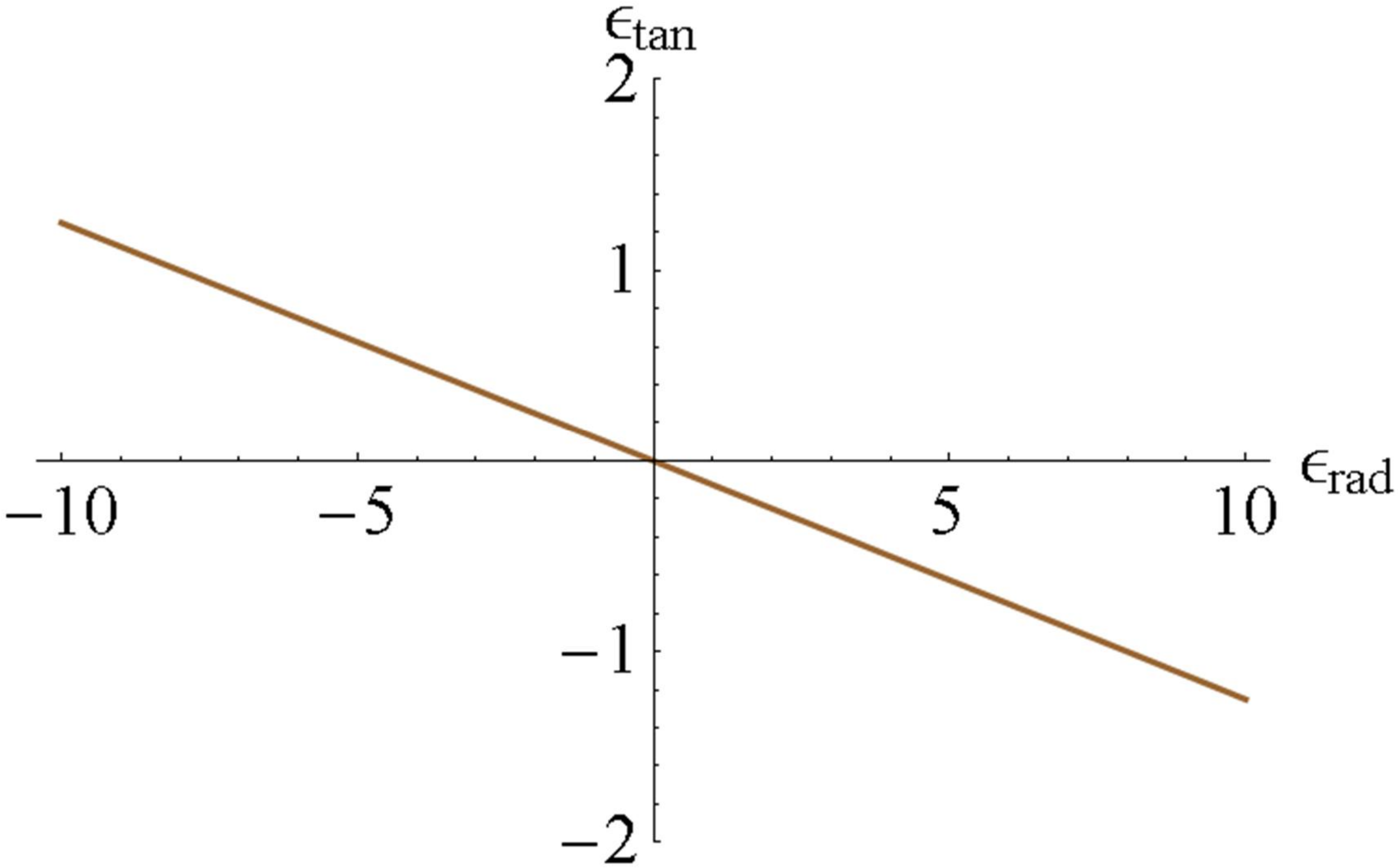


$$\alpha = 3\frac{\varepsilon - 1}{\varepsilon + 2}$$

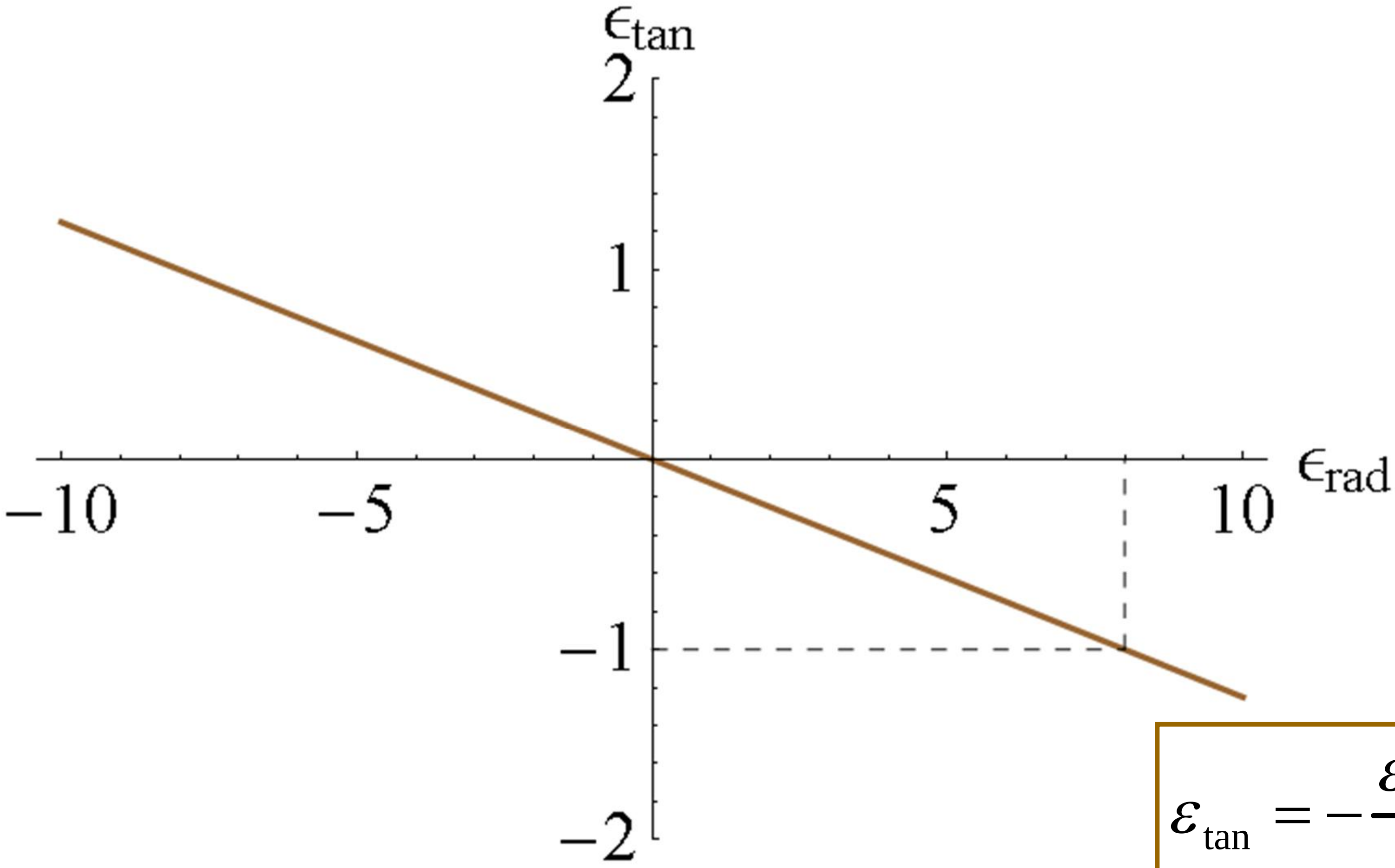
$$\alpha = 3\frac{\varepsilon_\mathrm{r} + 2 - \varepsilon_\mathrm{r}\sqrt{1 + 8\varepsilon_\mathrm{t}/\varepsilon_\mathrm{r}}}{\varepsilon_\mathrm{r} - 4 - \varepsilon_\mathrm{r}\sqrt{1 + 8\varepsilon_\mathrm{t}/\varepsilon_\mathrm{r}}}$$

T. Rimpiläinen, H. Wallén, H. Kettunen, A. Sihvola: Electrical response of systropic sphere, *IEEE TAP*, **60**(11), 5348-5355, 2012
K. Schulgasser: Sphere assemblage model for polycrystals and symmetric materials, *J. Appl. Phys.*, **54**(3), 1380-1382, 1983

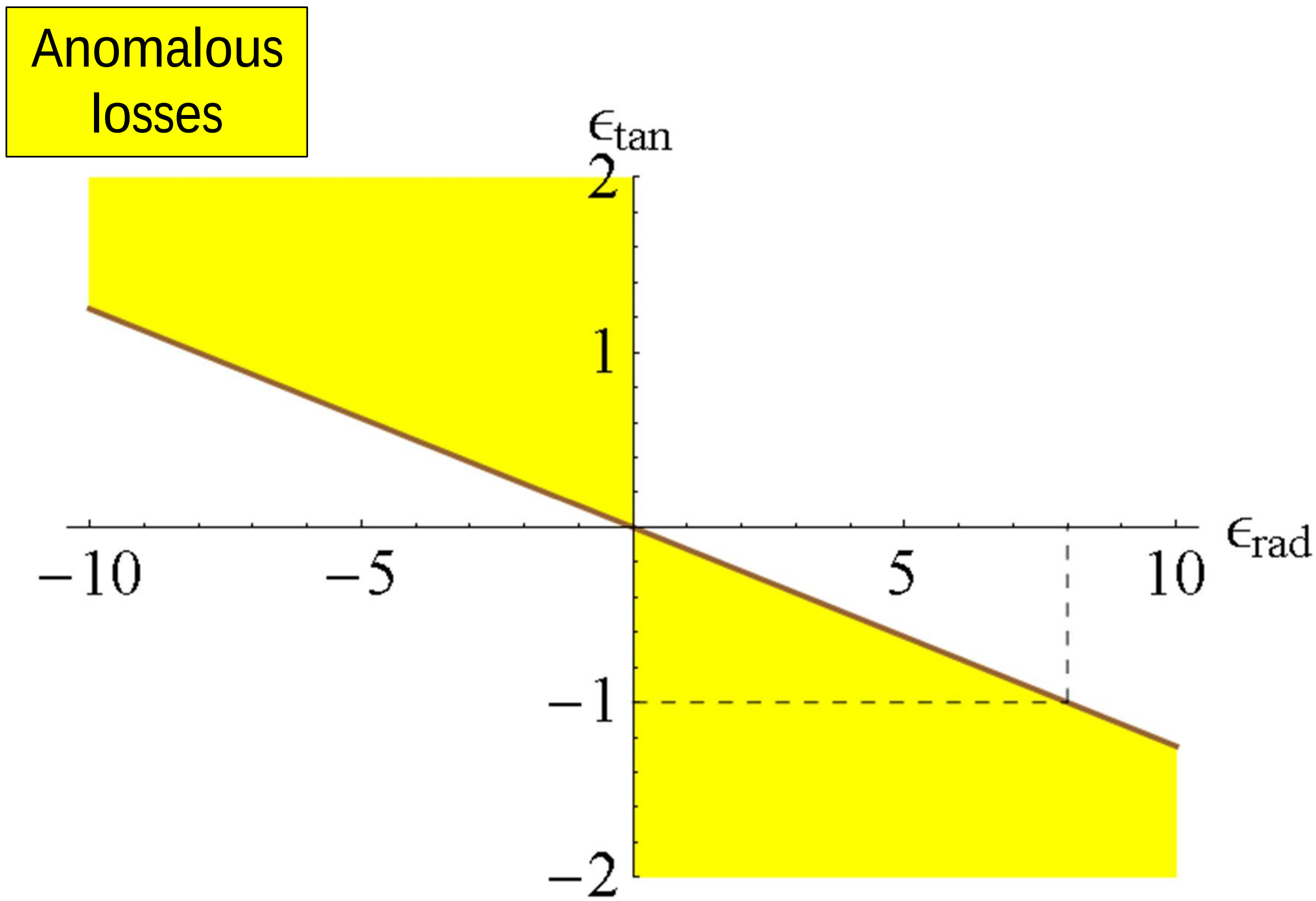

ε_{tan}
ε_{rad}
hyperbolic
(indefinite)
DPS
DNG
hyperbolic
(indefinite)



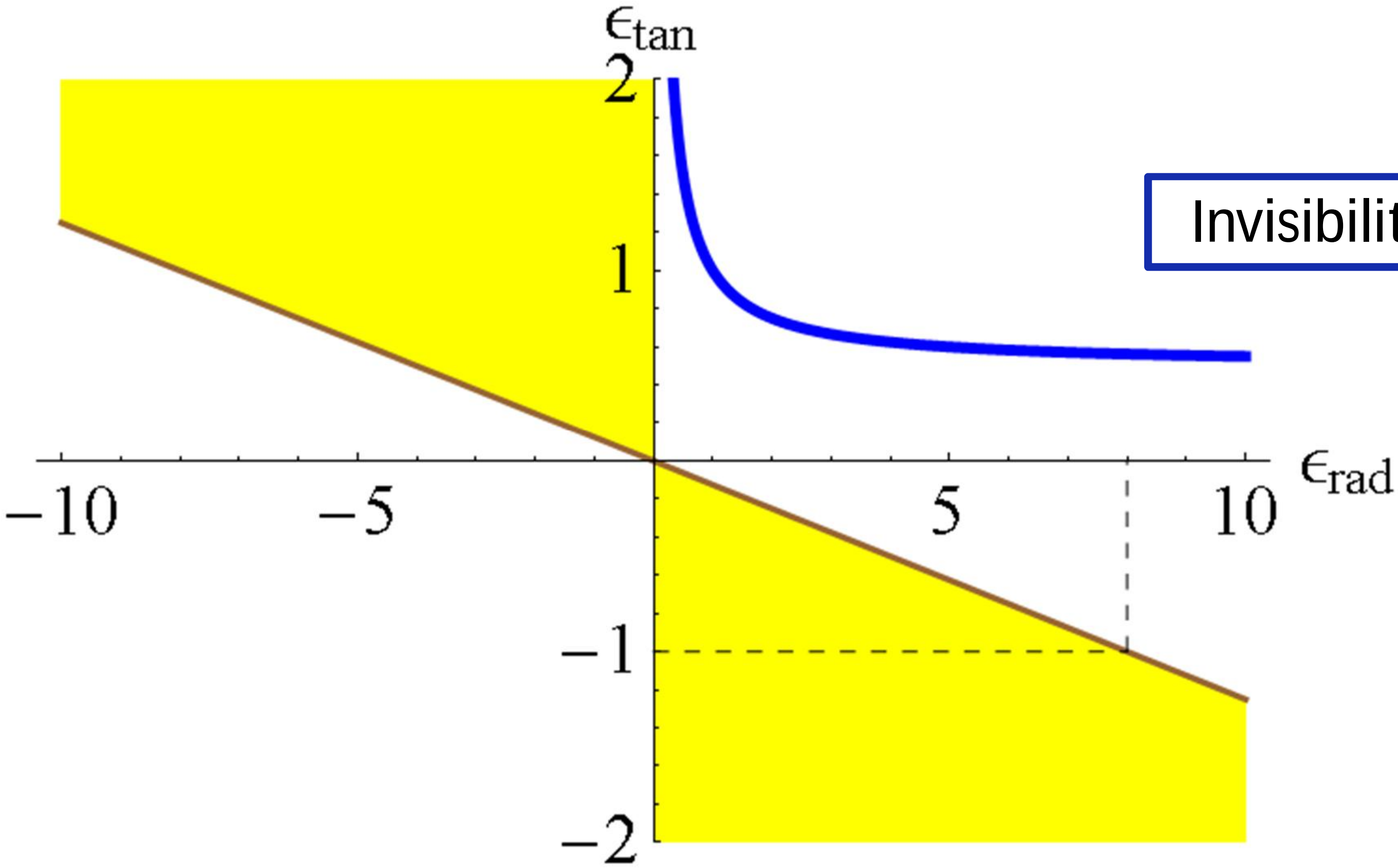
ϵ_{tan}
2
1
−1
−2
ϵ_{rad}
−10
−5
5
10



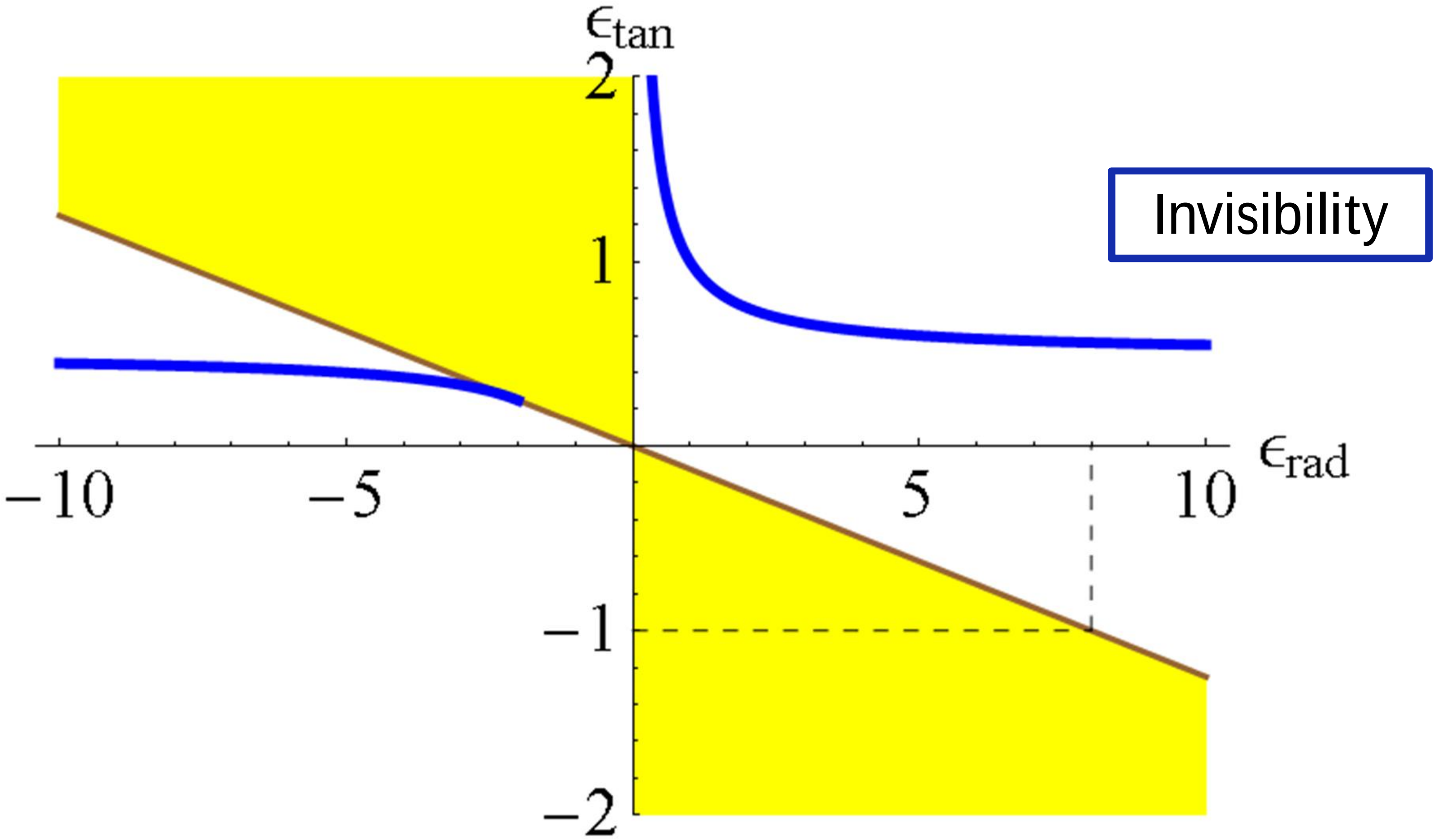
$$\varepsilon_{tan} = -\frac{\varepsilon_{rad}}{8}$$



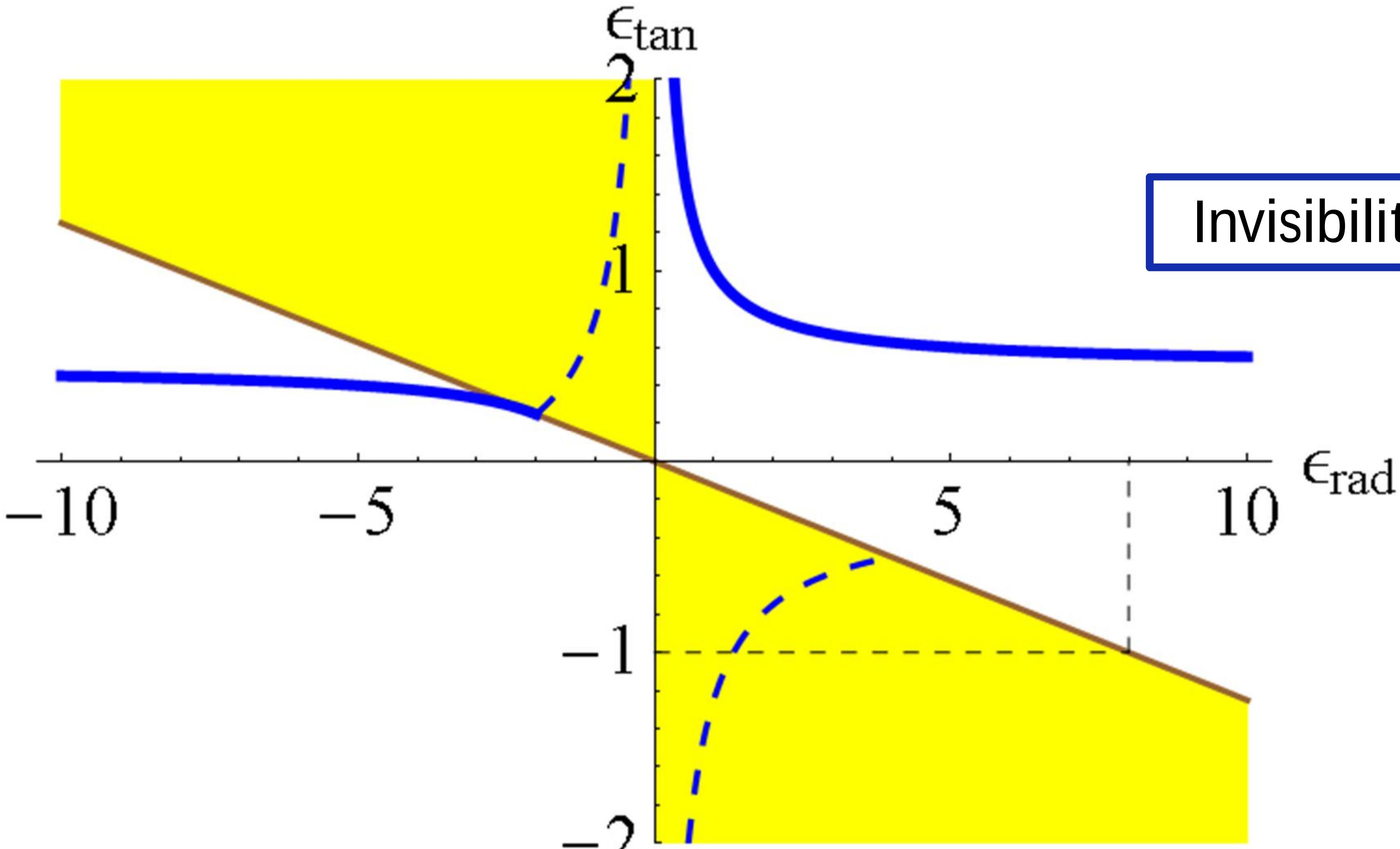
Anomalous
losses
ϵ_{tan}
2
1
−1
−2
−10
−5
5
10
ϵ_{rad}



Invisibility

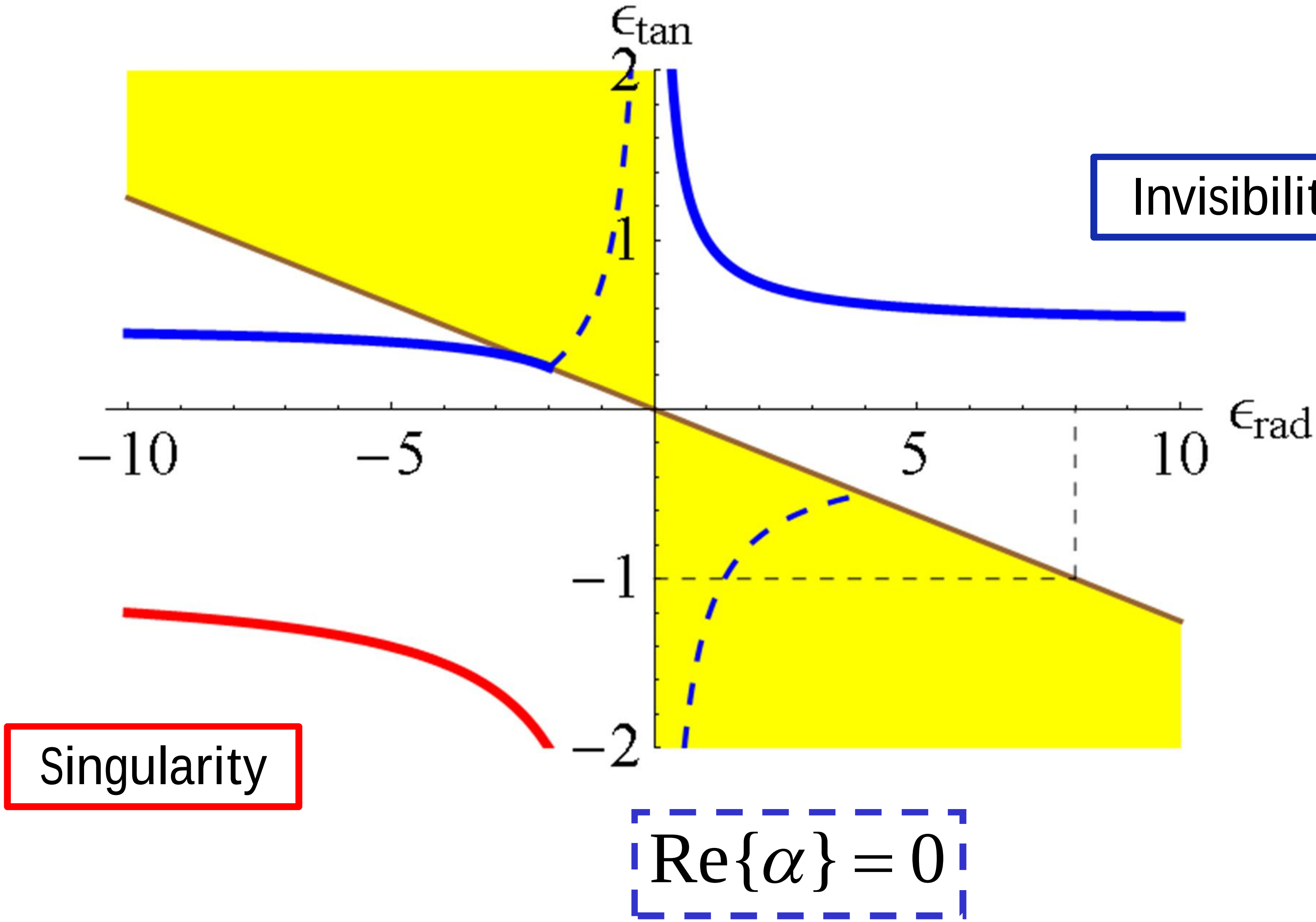


ϵ_{tan}
2
1
−1
−2
−10
−5
5
10
ϵ_{rad}
Invisibility

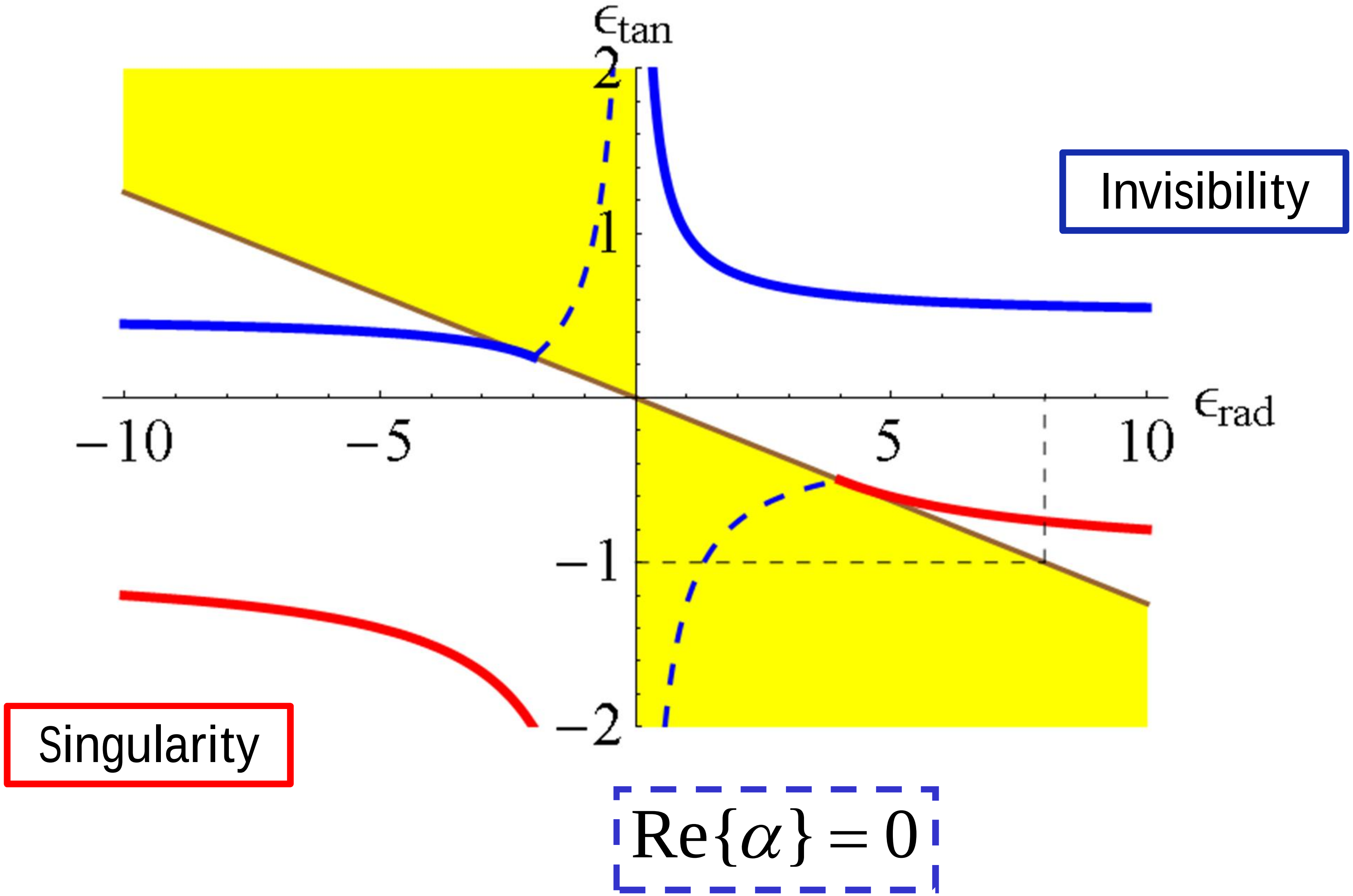


Invisibility

$\mathrm{Re}\{\alpha\} = 0$

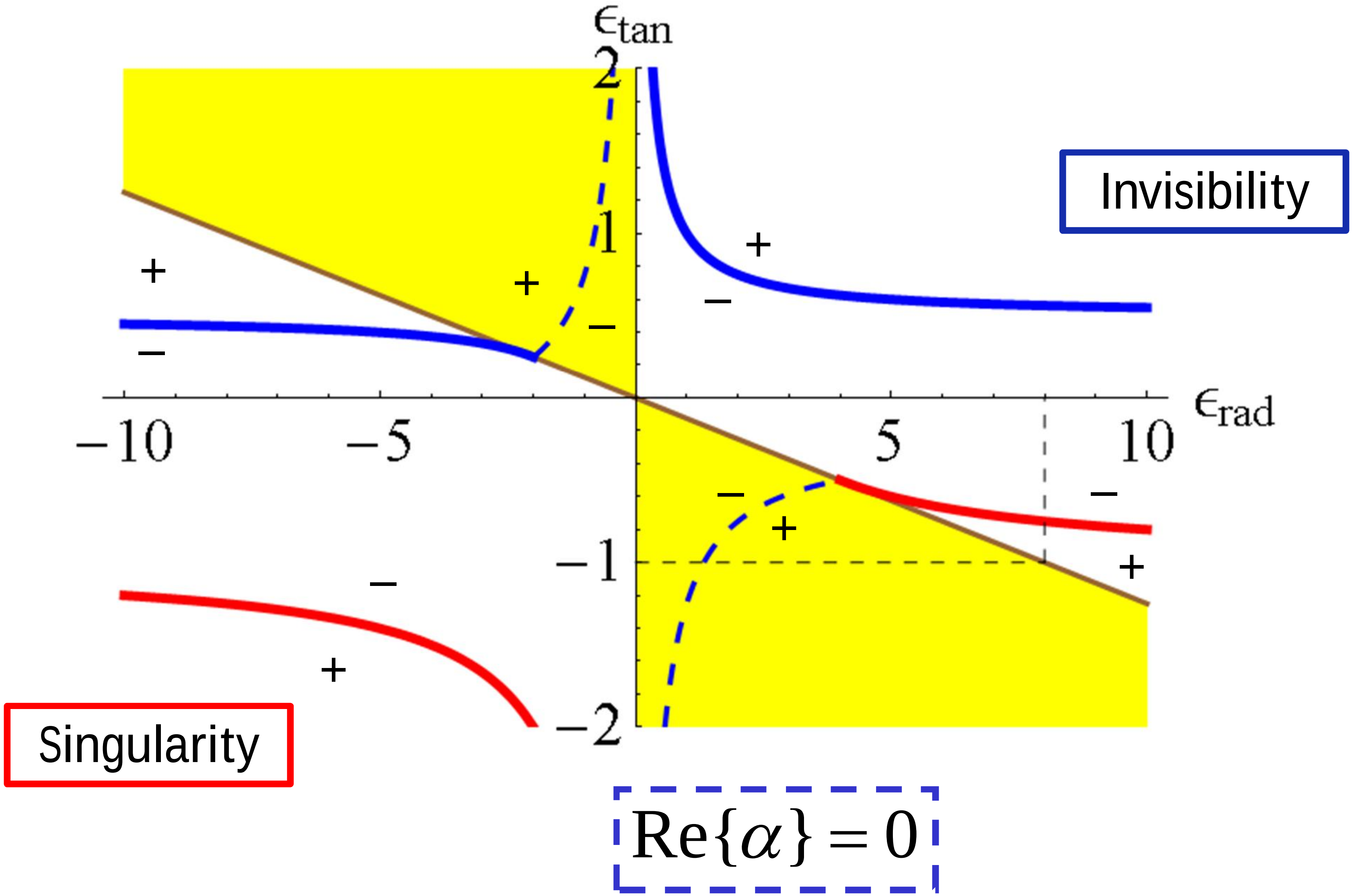


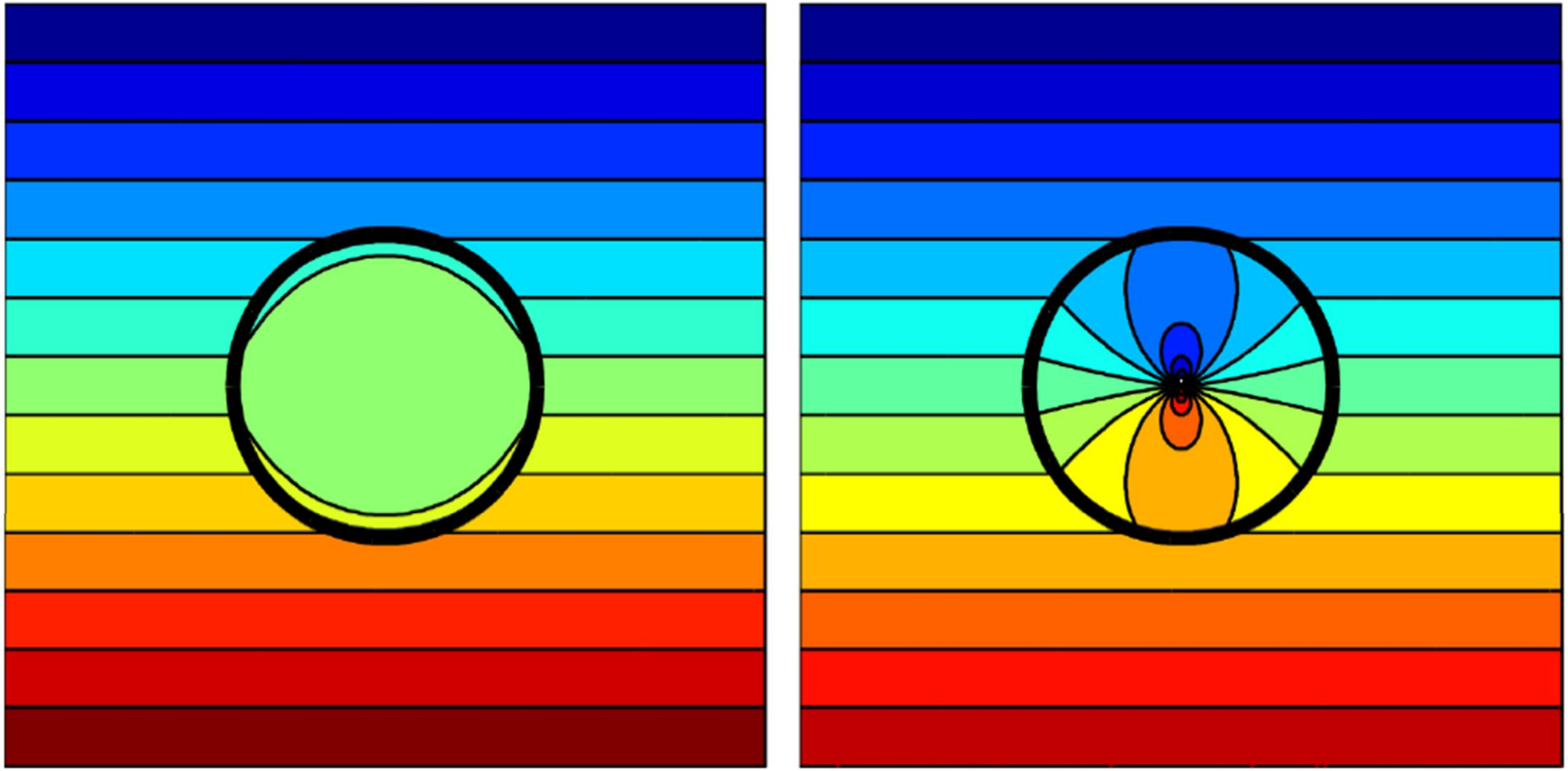
ϵ_{tan}
2
1
−1
−2
−10
−5
5
10
ϵ_{rad}
Invisibility
Singularity
$\mathrm{Re}\{\alpha\} = 0$



ϵ_{tan}
2
1
−1
−2
ϵ_{rad}
−10
−5
5
10
Invisibility
Singularity
$\mathrm{Re}\{\alpha\} = 0$

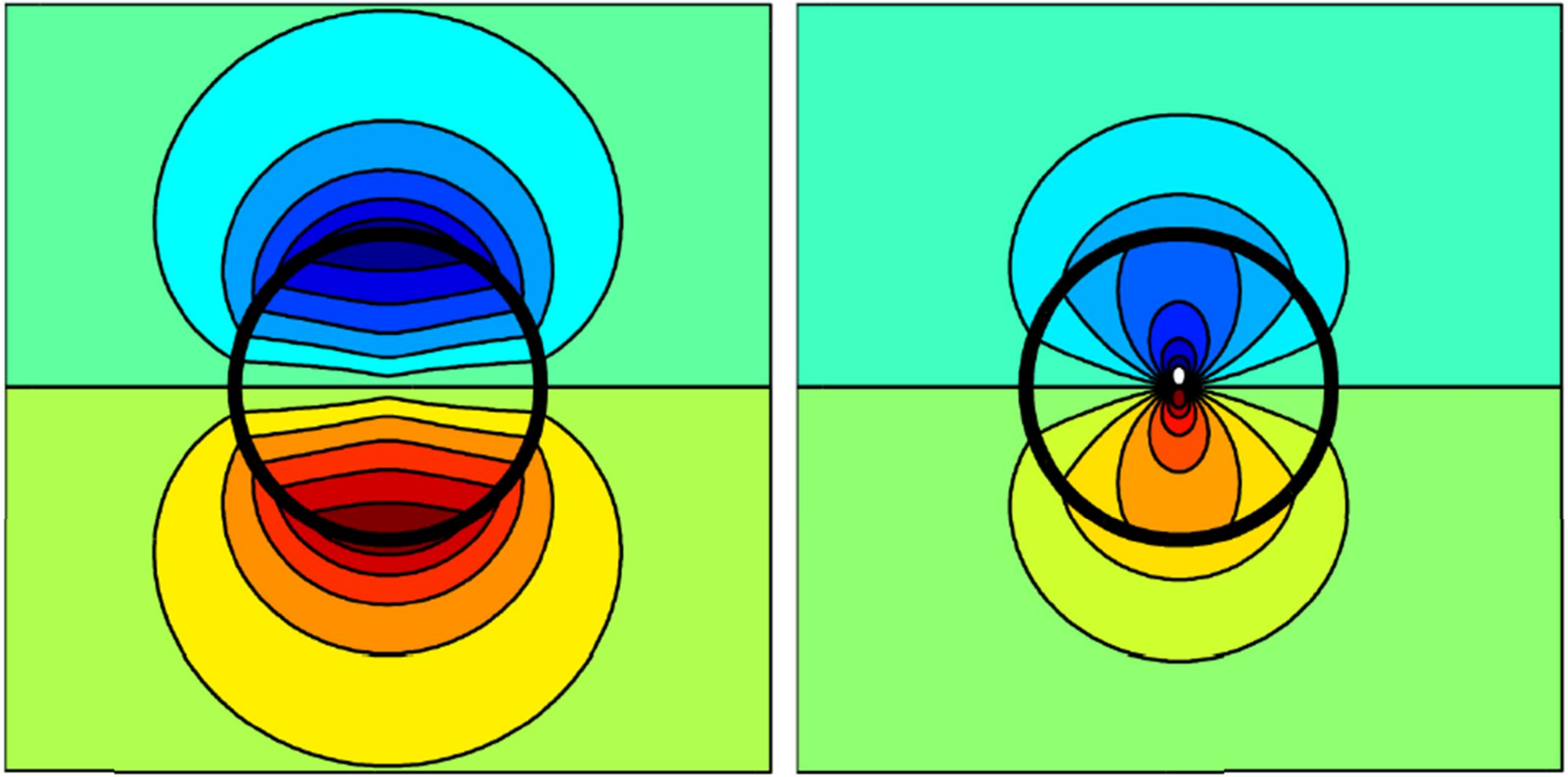
Sign $\pm$ of the (real part) of α





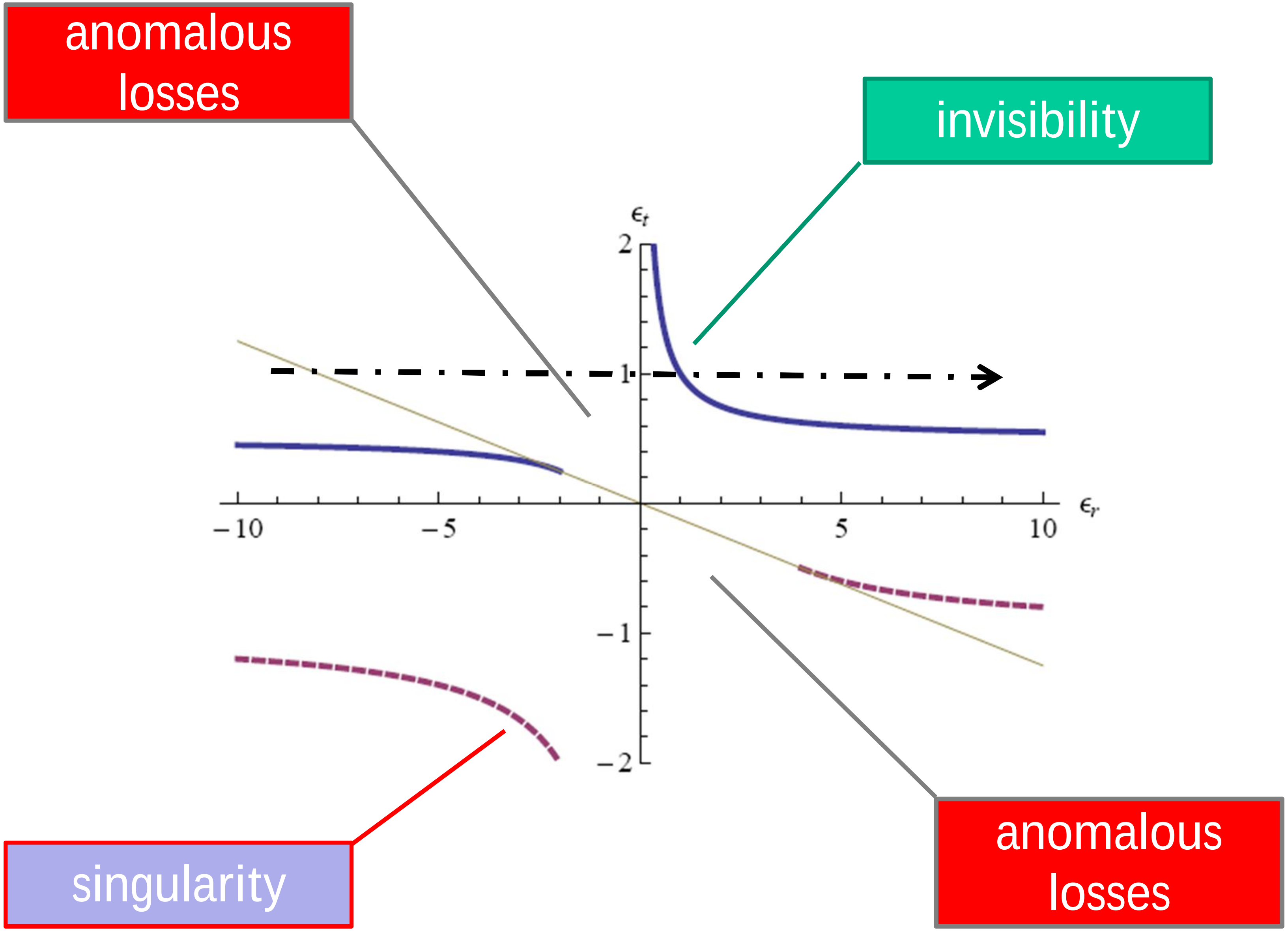
$$\varepsilon_{\text{rad}} = 0.1 \qquad \varepsilon_{\tan} = 5.5$$

$$\varepsilon_{\text{rad}} = -3 \qquad \varepsilon_{\tan} = 1/3$$



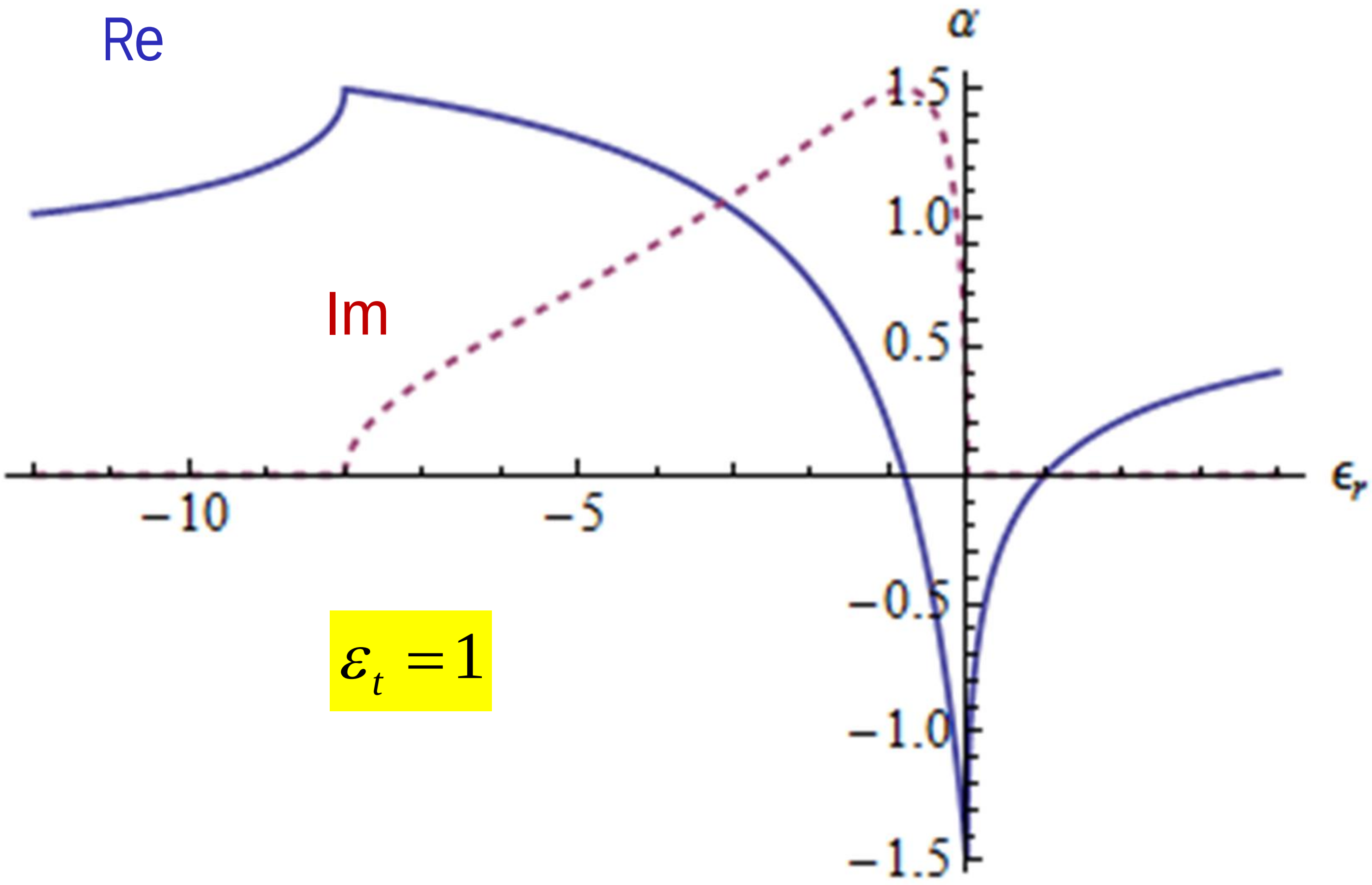
$$\varepsilon_{\text{rad}} = -3 \qquad \varepsilon_{\tan} = -5/3$$

$$\varepsilon_{\text{rad}} = +5 \qquad \varepsilon_{\tan} = -3/5$$



anomalous losses
invisibility
ϵ_i
2
1
−1
−2
−10
−5
5
10
ϵ_r
singularity
anomalous losses

RU sphere: complex polarizability

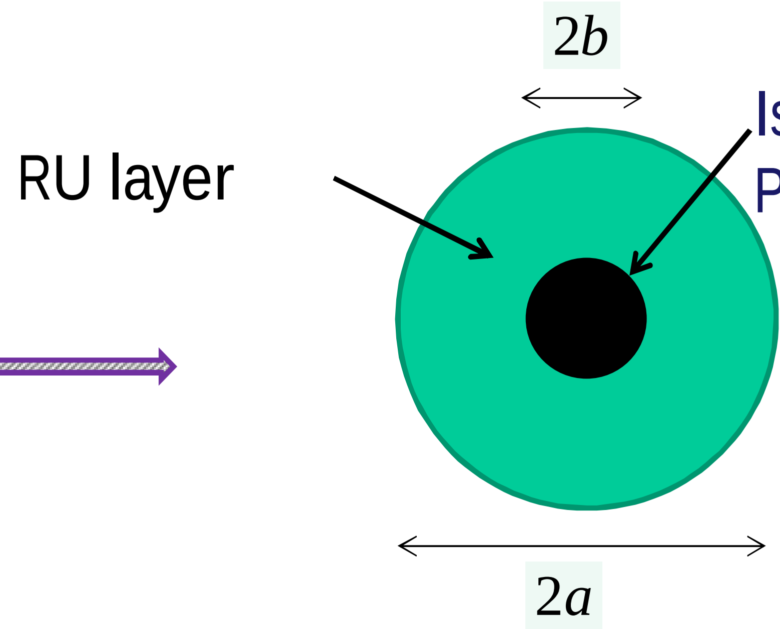


Intact RA sphere

- is an idealization
- but is it a true limit of a realistic RA sphere?

Undefined origin?

$$\alpha_{\mathrm{int}} = 3\frac{\varepsilon_{\mathrm{r}} + 2 - \varepsilon_{\mathrm{r}}\sqrt{1+8\varepsilon_{\mathrm{t}}/\varepsilon_{\mathrm{r}}}}{\varepsilon_{\mathrm{r}} - 4 - \varepsilon_{\mathrm{r}}\sqrt{1+8\varepsilon_{\mathrm{t}}/\varepsilon_{\mathrm{r}}}}$$



$$C \to 0$$

$$\alpha \to \alpha_{\mathrm{int}}$$

$$\alpha = 3\frac{\varepsilon_{\mathrm{r}} + 2 - \varepsilon_{\mathrm{r}}\sqrt{\cdot} - C}{\varepsilon_{\mathrm{r}} - 4 - \varepsilon_{\mathrm{r}}\sqrt{\cdot} - C}$$

$$C = \left(\varepsilon_{\mathrm{r}} + 2 + \varepsilon_{\mathrm{r}}\sqrt{\cdot}\right)\left(\frac{b}{a}\right)^{\sqrt{\cdot}}$$

$$\sqrt{\cdot} = \sqrt{1+8\varepsilon_{\mathrm{t}}/\varepsilon_{\mathrm{r}}}$$

Limit towards the intact sphere?

$$C = \left(\varepsilon_{\mathrm{r}} + 2 + \varepsilon_{\mathrm{r}}\sqrt{\cdot}\right)\left(\frac{b}{a}\right)^{\sqrt{\cdot}}$$

$$\sqrt{\cdot} = \sqrt{1 + 8\varepsilon_{\mathrm{t}} / \varepsilon_{\mathrm{r}}}$$

$$\nu = \frac{1}{2}(-1 + \sqrt{\cdot})$$

$$C \to 0$$

$$\alpha \to \alpha_{\mathrm{int}} \,(= \alpha_{\mathrm{RU}})$$

$$\boxed{\frac{\varepsilon_{\mathrm{t}}}{\varepsilon_{\mathrm{r}}} > -1/8}$$

$$\sqrt{\cdot} = 2\nu + 1 \,(\mathrm{real}) \Rightarrow \lim_{b/a \to 0}\left(\frac{b}{a}\right)^{\sqrt{\cdot}} = 0$$

$$\boxed{\alpha \to \alpha_{\mathrm{RU}}}$$

$$\boxed{\frac{\varepsilon_{\mathrm{t}}}{\varepsilon_{\mathrm{r}}} < -1/8}$$

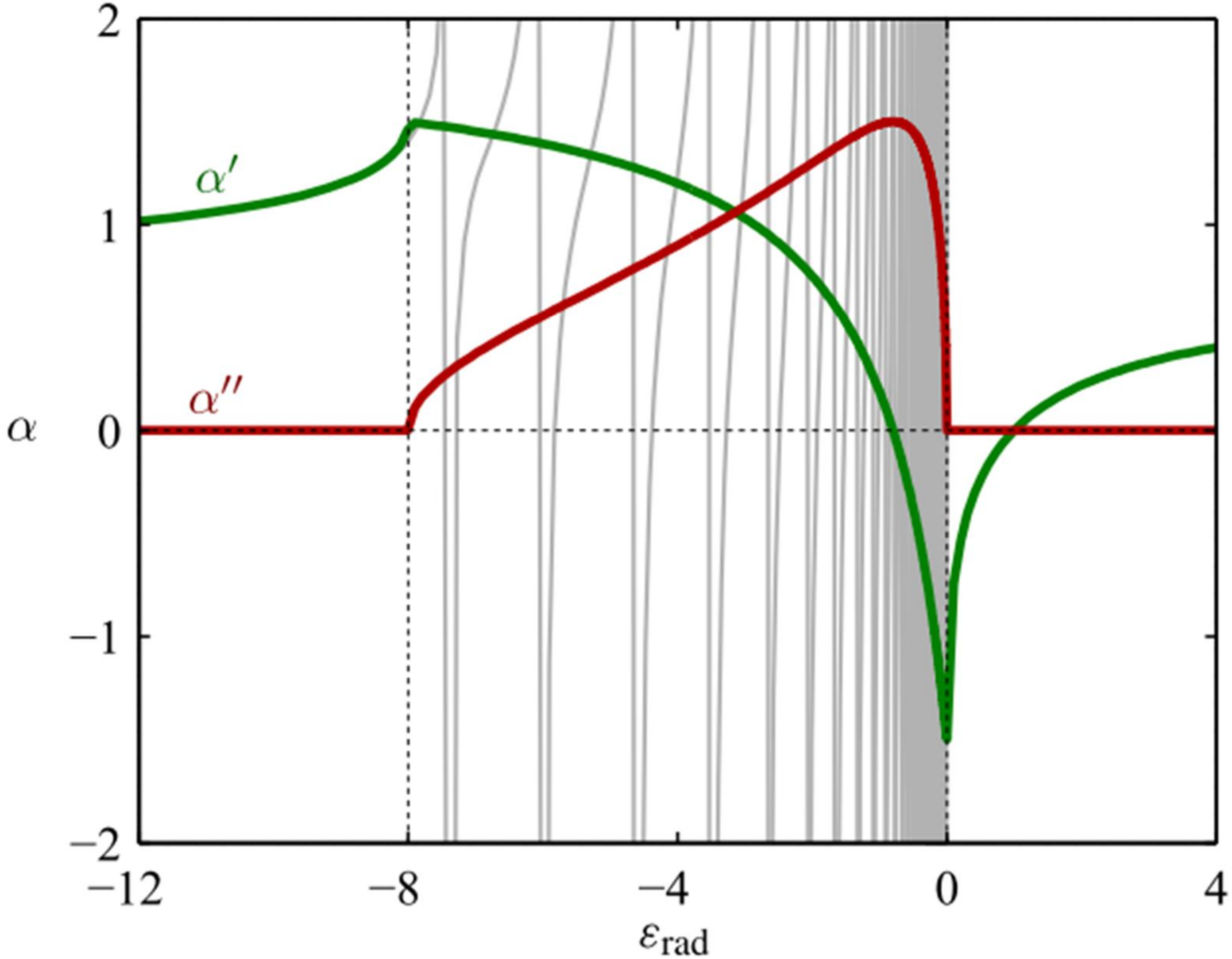
$$\sqrt{\cdot} = \sqrt{1 + 8\varepsilon_{\mathrm{t}} / \varepsilon_{\mathrm{r}}} = \mathrm{j}\beta$$

$$\left(\frac{b}{a}\right)^{\sqrt{\cdot}} = \left(\frac{b}{a}\right)^{\mathrm{j}\beta} = \cos\left(\beta \log\frac{b}{a}\right) + \mathrm{j}\sin\left(\beta \log\frac{b}{a}\right)$$

limit $b/a \to 0$ does not exist!

$\boxed{\frac{\varepsilon_{\mathrm{t}}}{\varepsilon_{\mathrm{r}}} < -1/8}$ + small losses → $\mathrm{Re}\{\sqrt{\cdot}\} \neq 0 \;\Rightarrow\; \lim_{b/a \to 0}\left(\frac{b}{a}\right) = 0$

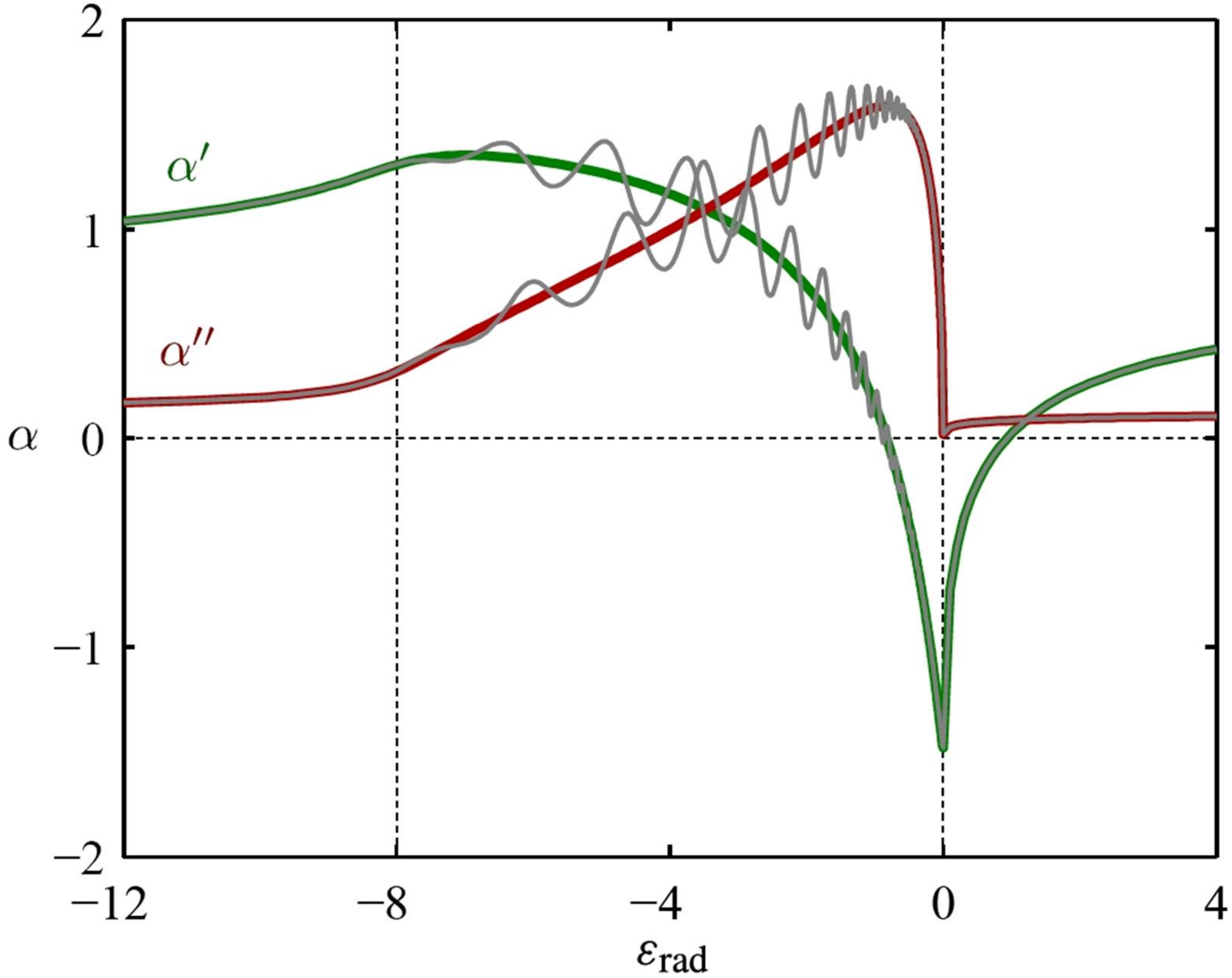
$$\boxed{\alpha \to \alpha_{\mathrm{RU}}\,!}$$



$\varepsilon_{\tan} = 1$

$b/a = 10^{-10}$

Wallén, Kettunen, and Sihvola, "Anomalous absorption, plasmonic resonances, and invisibility of radially anisotropic spheres," *Radio Science,* 50(1)18-28, 2015.



$$\varepsilon_{\tan} = 1 - \mathrm{j}\,0.1$$

$$b/a = 10^{-10}$$

Wallén, Kettunen, and Sihvola, “Anomalous absorption, plasmonic resonances, and invisibility of radially anisotropic spheres,” *Radio Science,* , 50(1)18-28, 2015.

$$\lim_{\text{losses}\to 0}\left(\lim_{\text{core size}\to 0}\alpha\right) \neq \lim_{\text{core size}\to 0}\left(\lim_{\text{losses}\to 0}\alpha\right)$$

- The original result (the intact RA sphere polarizability) is a faithful limit (for all values of the permittivity components) of the punctured RA sphere
  - provided that finite intrinsic losses (arbitrarily small) are present
- Hence: instead of ”emergence of loss”
  - ”super-enhancement of loss”

From quasistatics towards dynamics…

- How regular is the limit $\omega \rightarrow 0$?
- Static polarizability → Rayleigh scattering
- Dynamics: Mie scattering
  - Mie series obviously more complicated than in the isotropic sphere case…

$$Q_{\mathrm{sca}} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) \left(|a_n|^2 + |b_n|^2 \right)$$

$$Q_{\mathrm{ext}} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) \, \mathrm{Re}\{a_n + b_n\}$$

$$Q_{\mathrm{abs}} = Q_{\mathrm{ext}} - Q_{\mathrm{sca}}$$

$$\psi_\nu(x) = x j_\nu(x)$$

$$\xi_\nu(x) = x h_\nu^{(2)}(x)$$

$$m = \sqrt{\varepsilon_{\mathrm{t}}}$$

$$a_n = \frac{m\psi_\nu(mx)\psi_n'(x) - \psi_n(x)\psi_\nu'(mx)}{m\psi_\nu(mx)\xi_n'(x) - \xi_n(x)\psi_\nu'(mx)}$$

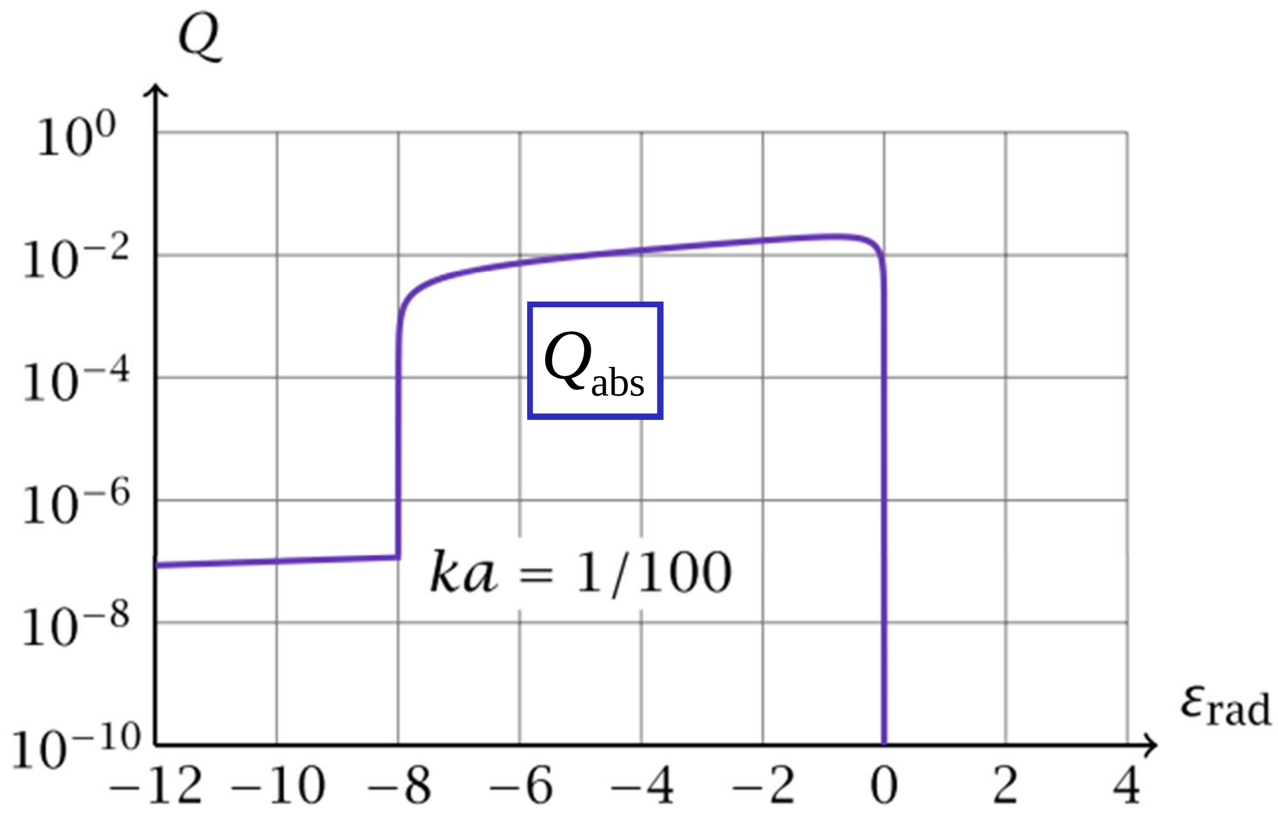
$$b_n = \frac{\psi_n(mx)\psi_n'(x) - m\psi_n(x)\psi_n'(mx)}{\psi_n(mx)\xi_n'(x) - m\xi_n(x)\psi_n'(mx)}$$

$$\nu = \nu(n) = -\frac{1}{2} + \sqrt{n(n+1)\frac{\varepsilon_{\mathrm{t}}}{\varepsilon_{\mathrm{r}}} + \frac{1}{4}}$$

```
>> nu=1.2;   x=3.4;
>> js = sqrt(pi ./(2* x)) * besselj(nu + 0.5, x)

js =

    0.2949

>> nu=1.2+i;   x=3.4;
>> js = sqrt(pi ./(2* x)) * besselj(nu + 0.5, x)
Error using besselj
NU must be real.
```

$$\boxed{j_\nu(x)}$$

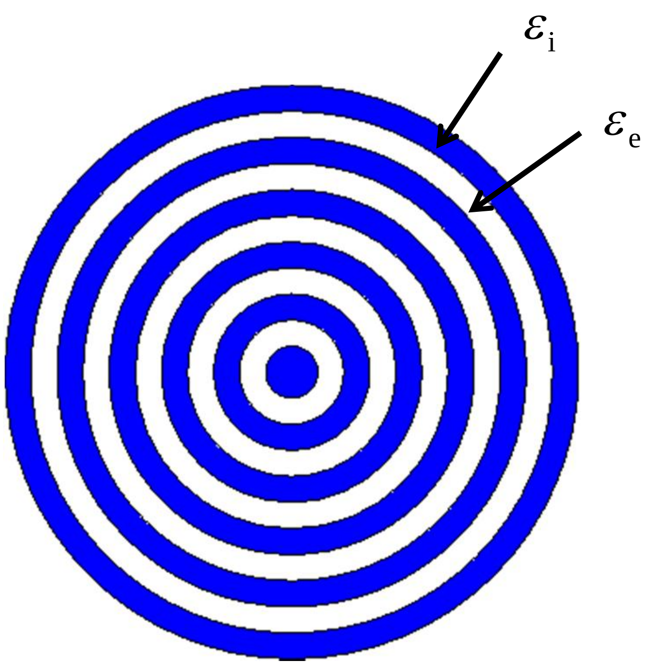
```
In[67]:= SphericalBesselJ[1.2, 3.4]

Out[67]= 0.294941

In[68]:= SphericalBesselJ[1.2 + I, 3.4]

Out[68]= 0.483046 + 0.102482 i
```



$\varepsilon_{\tan} = 1$

Wallén, Kettunen, and Sihvola, "Anomalous absorption, plasmonic resonances, and invisibility of radially anisotropic spheres," *Radio Science,* 15(1)18-28, 2015.

RA sphere from the onion structure

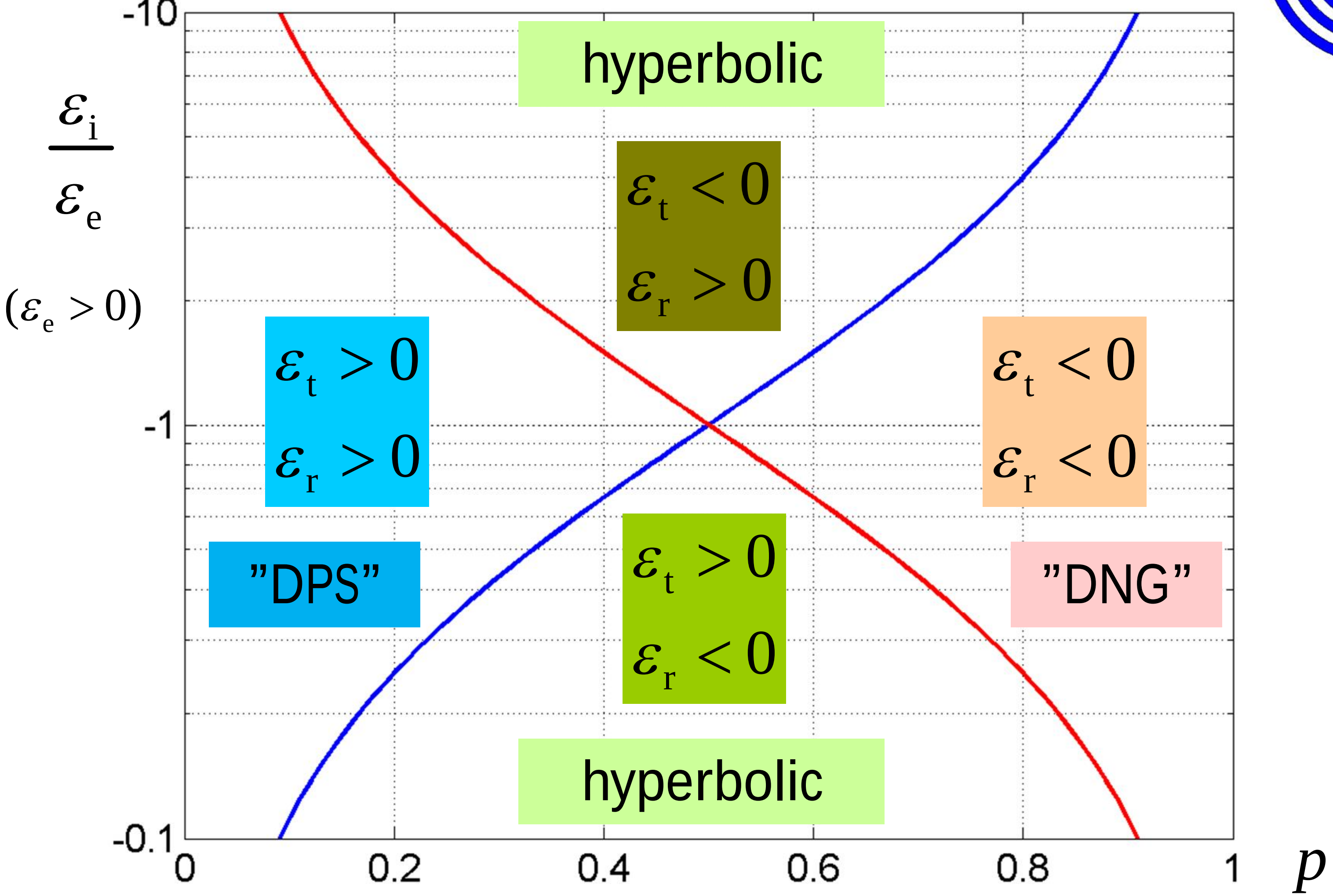


$$\varepsilon_t = p\varepsilon_i + (1-p)\varepsilon_e$$

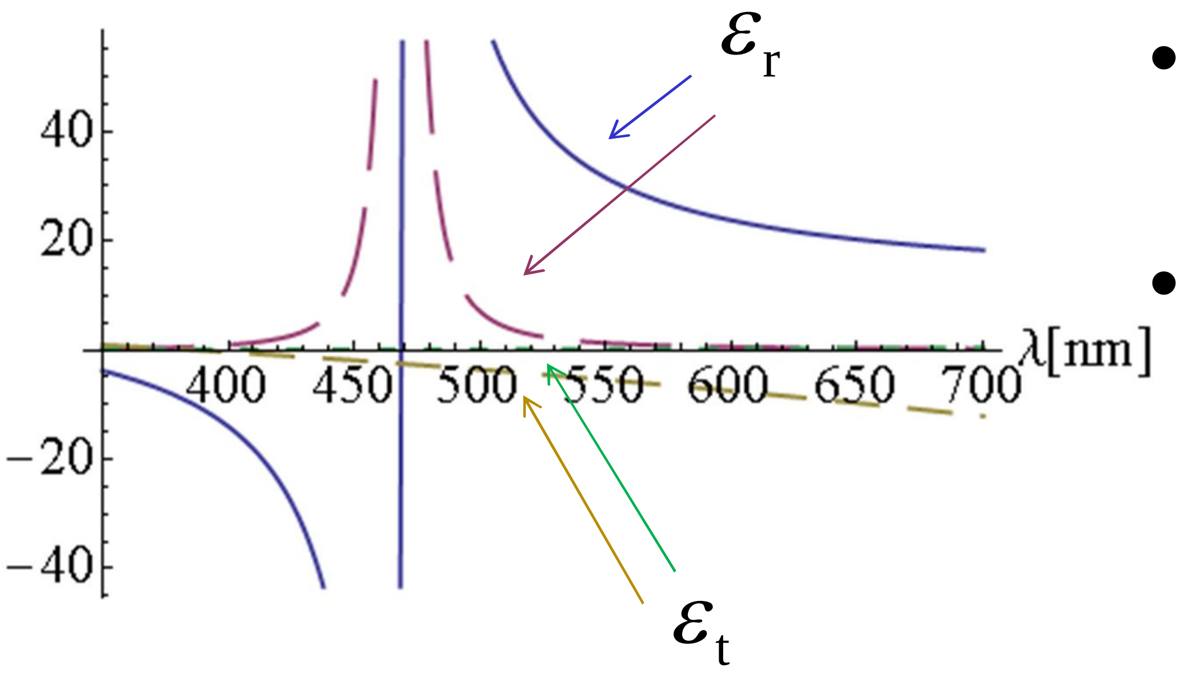
$$\varepsilon_r = \frac{1}{p/\varepsilon_i + (1-p)/\varepsilon_e}$$

$$p = \frac{d_i}{d_i + d_e}$$

layer thicknesses d_i , d_e


-10
-1
-0.1
0
0.2
0.4
0.6
0.8
1
p
εi / εe
(εe > 0)
hyperbolic
εt < 0
εr > 0
εt > 0
εr > 0
εt < 0
εr < 0
"DPS"
εt > 0
εr < 0
"DNG"
hyperbolic

Hyperbolic/indefinite onion structure



- indefinite below 386 nm
- negative definite: 369 ... 469 nm
- indefinite above 469 nm

$\varepsilon_i = \mathrm{Ag}$

$\varepsilon_e = 5$

$p = 0.6$

... back to simple mixtures...

History of homogenization:
”a play in five acts”

- Maxwell Garnett (the mixing principle)
- Bruggeman (effective medium idea)
- Beran, Hashin, Shtrikman, Bergman (bounding and limits)
- Percolation (precursor of emergence)
- Computational possibilities (brute-force)
- 6th scene (?): metamaterials paradigm

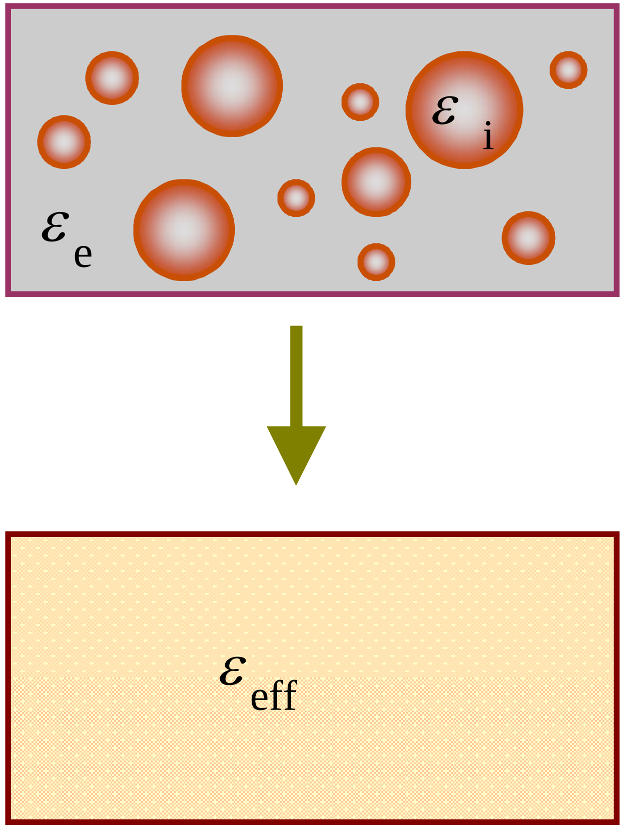
Chr. Brosseau, *J. Phys. D: Appl. Phys.* **39** (2006), 1277-94**.** **A. Sihvola:** *Phot. Nanostr. Fund. Appl.* **11** (2013), 364-73.

Maxwell Garnett mixing formula

$$\langle \mathbf{D} \rangle = \varepsilon_{\mathrm{eff}} \langle \mathbf{E} \rangle$$

$$\langle \mathbf{E} \rangle = f\,\mathbf{E}_{\mathrm{i}} + (1-f)\,\mathbf{E}_{\mathrm{e}}$$

$$\langle \mathbf{D} \rangle = f\,\varepsilon_{\mathrm{i}}\mathbf{E}_{\mathrm{i}} + (1-f)\,\varepsilon_{\mathrm{e}}\mathbf{E}_{\mathrm{e}}$$



$$\mathbf{E}_{\mathrm{i}} = \frac{3\varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{i}} + 2\varepsilon_{\mathrm{e}}}\mathbf{E}_{\mathrm{e}}$$

$$\varepsilon_{\mathrm{eff}} = \varepsilon_{\mathrm{e}} + 3f\,\varepsilon_{\mathrm{e}}\frac{\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{i}} + 2\varepsilon_{\mathrm{e}} - f(\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}})}$$

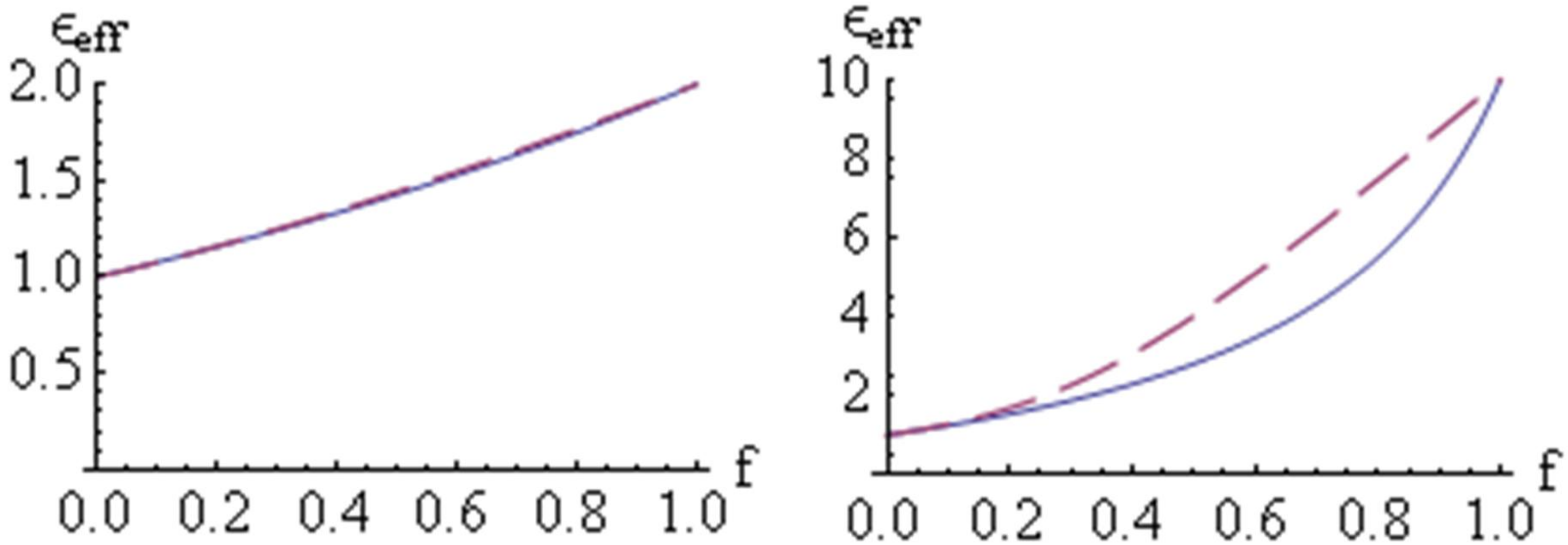
J. C. Maxwell Garnett. Colours in metal glasses and metal films. *Trans. the Royal Society (London), 203:385–420, 1904.*

Maxwell Garnett

$$\frac{\varepsilon_{\mathrm{eff}}-\varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{eff}}+2\varepsilon_{\mathrm{e}}}=f\,\frac{\varepsilon_{\mathrm{i}}-\varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{i}}+2\varepsilon_{\mathrm{e}}}$$

Bruggeman (symmetric)

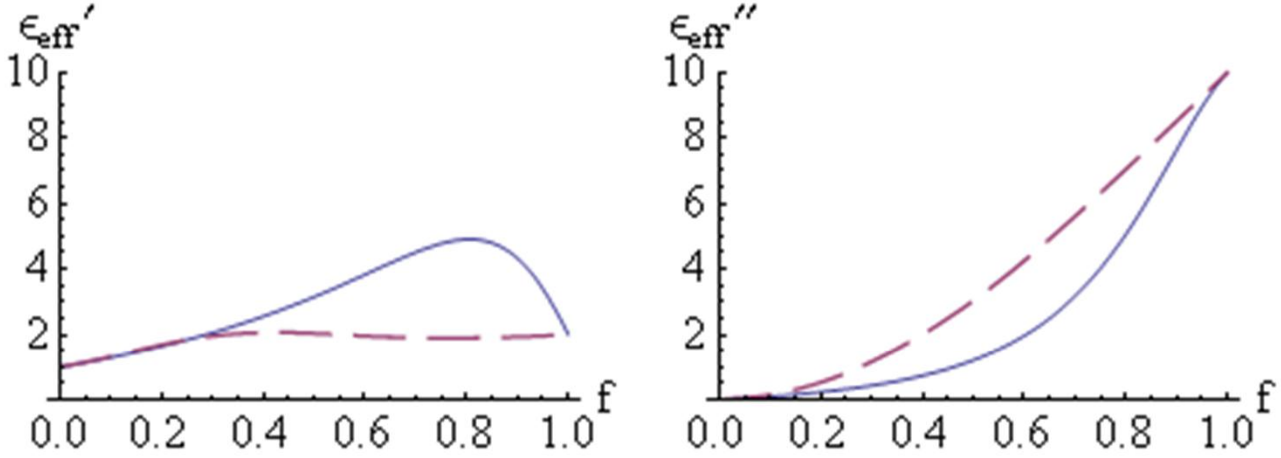
$$(1-f)\frac{\varepsilon_{\mathrm{e}}-\varepsilon_{\mathrm{eff}}}{\varepsilon_{\mathrm{e}}+2\varepsilon_{\mathrm{eff}}}+f\,\frac{\varepsilon_{\mathrm{i}}-\varepsilon_{\mathrm{eff}}}{\varepsilon_{\mathrm{i}}+2\varepsilon_{\mathrm{eff}}}=0$$



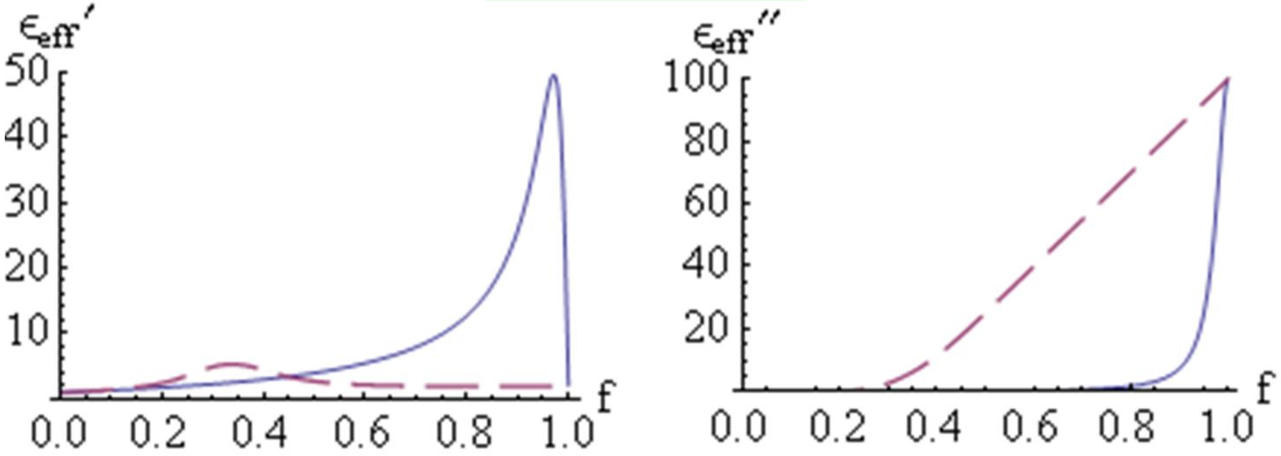
Maxwell Garnett

Bruggeman

$$\varepsilon_i = 2 - j \cdot 10$$



$$\varepsilon_i = 2 - j \cdot 100$$



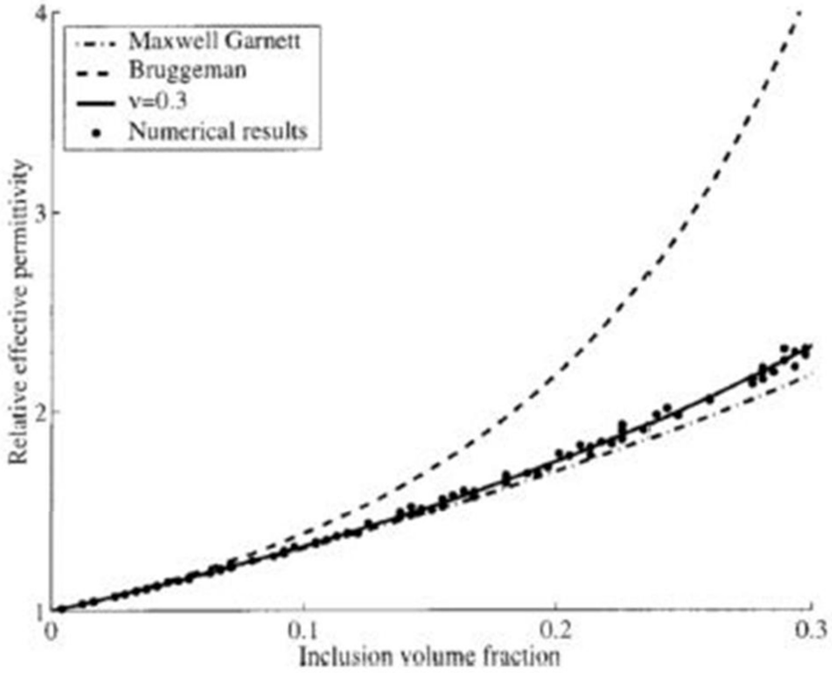
Maxwell Garnett

Bruggeman

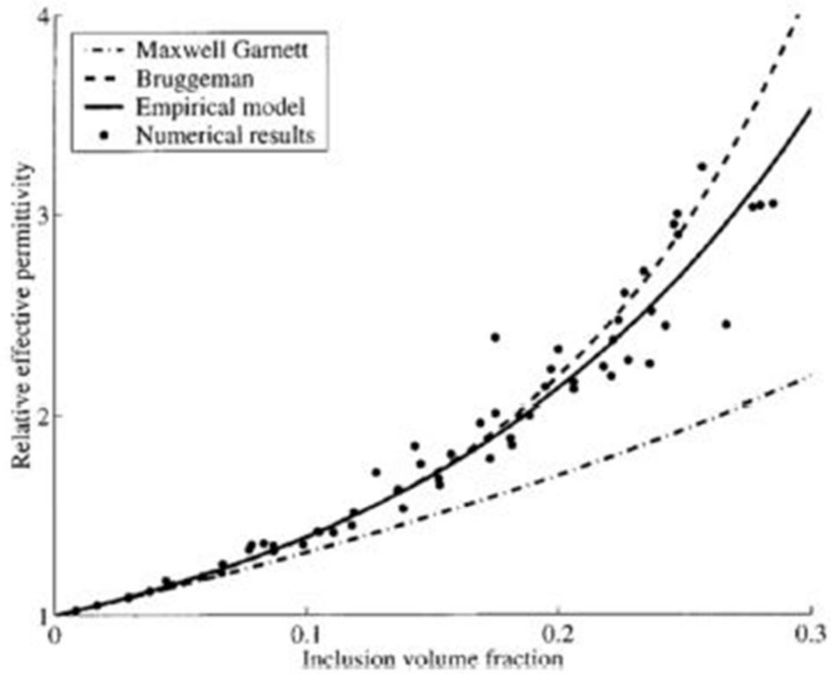
Real lossless composites

Numerical results for random mixtures with $\varepsilon_i/\varepsilon_e = 51$

Without clustering
⇒ near Maxwell Garnett

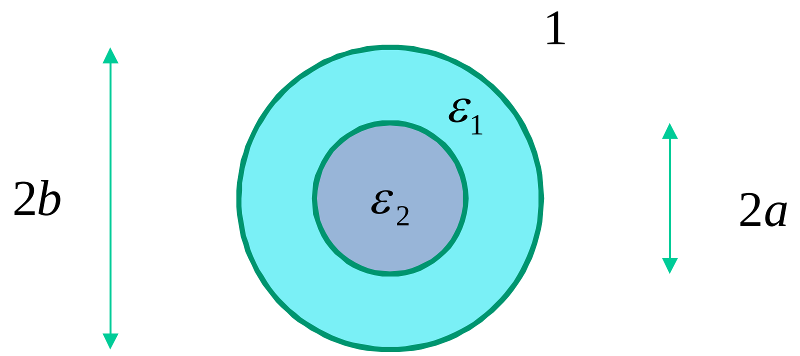


With clustering
⇒ near Bruggeman



Figures from: K. Kärkkäinen, A. Sihvola, and K. Nikoskinen, "Analysis of a three-dimensional dielectric mixture with finite difference method," *IEEE Trans. Geosci. Remote Sens.*, 39 (2001) 1013–1018.

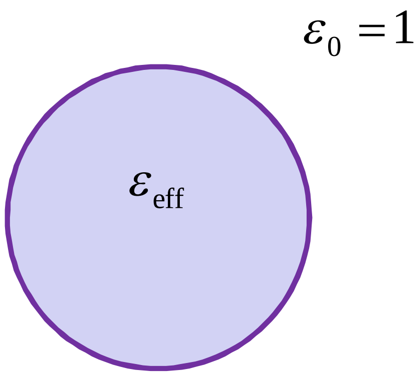
ANOTHER VIEW on Maxwell Garnett



$$\mathbf{p} = \alpha_{\mathrm{abs}}\mathbf{E}$$

$$\varepsilon_0 V\alpha$$

$$\alpha = 3\frac{(\varepsilon_1 - 1)(\varepsilon_2 + 2\varepsilon_1) + (a/b)^3(2\varepsilon_1 + 1)(\varepsilon_2 - \varepsilon_1)}{(\varepsilon_1 + 2)(\varepsilon_2 + 2\varepsilon_1) + 2(a/b)^3(\varepsilon_1 - 1)(\varepsilon_2 - \varepsilon_1)}$$



$$\alpha_{\text{eff}} = 3\frac{\varepsilon_{\text{eff}} - 1}{\varepsilon_{\text{eff}} + 2}$$

$$\alpha_{\text{eff}} = \alpha \quad \Longrightarrow \quad \varepsilon_{\text{eff}} = \varepsilon_1 + 3(a/b)^3 \varepsilon_1 \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + 2\varepsilon_1 - (a/b)^3(\varepsilon_2 - \varepsilon_1)}$$

Exactly Maxwell Garnett two-phase mixing formula !!!

Repercussions:

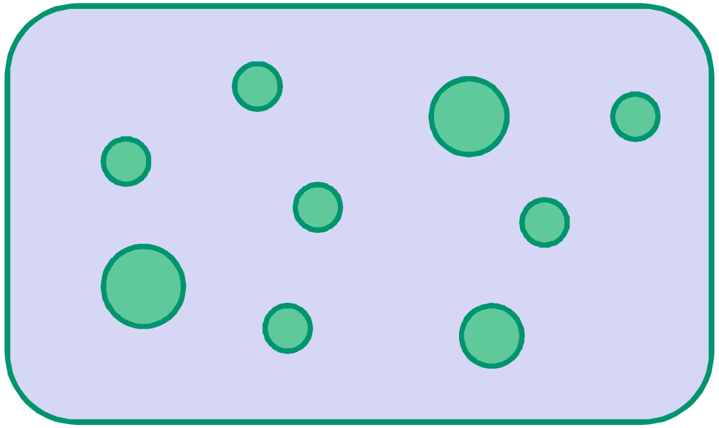
- Bounds for the polarizability
- Dispersion engineering
- Cloaking applications

Repercussions:

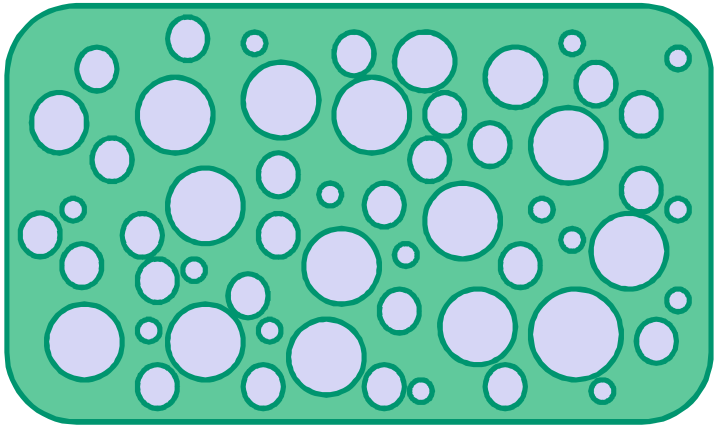
- Bounds for the polarizability
- Dispersion engineering
- Cloaking applications

MG and inverse MG

(MG non-symmetric)



Raisin pudding



Swiss cheese

$$\epsilon_i \rightarrow \epsilon_e$$
$$\epsilon_e \rightarrow \epsilon_i$$
$$p \rightarrow 1 - p$$

Hashin–Shtrikman bounds

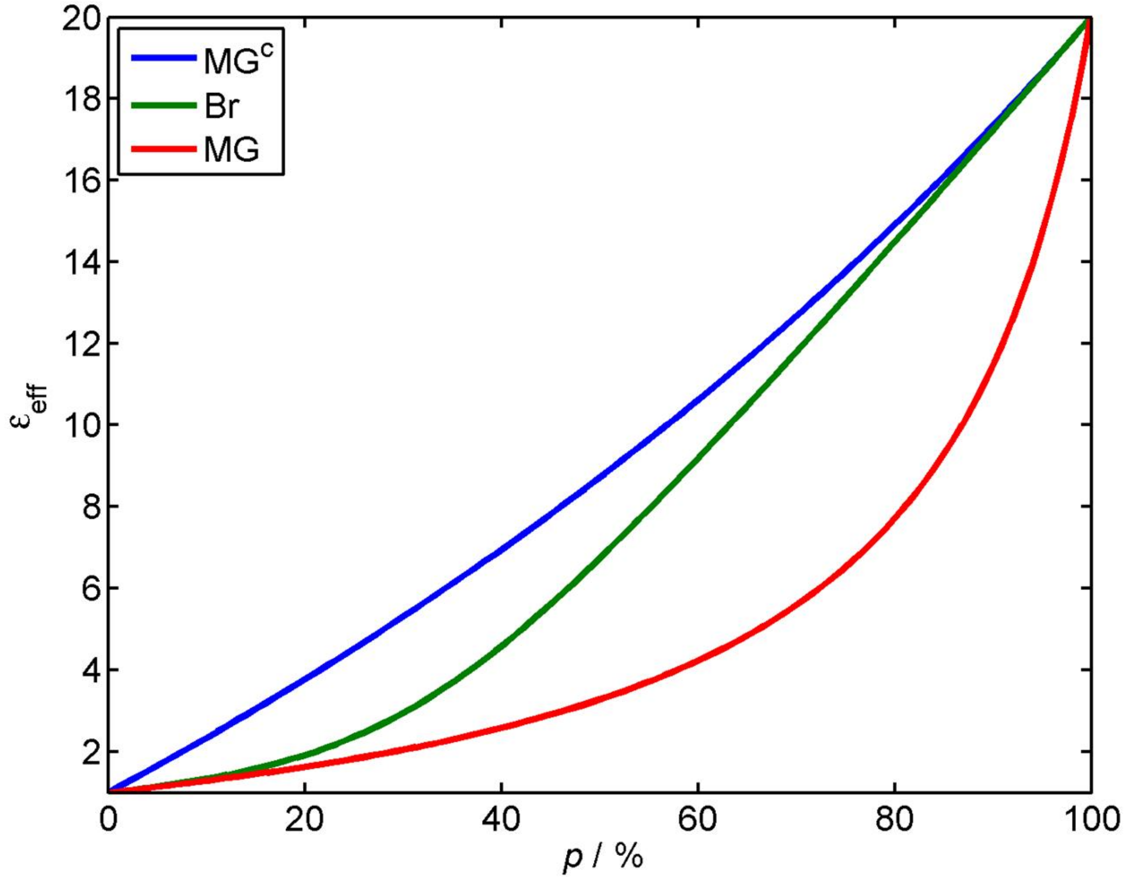
$$\boxed{\varepsilon_{\mathrm{MG}} \leq \varepsilon_{\mathrm{eff}} \leq \varepsilon_{\mathrm{invMG}}}$$

for ”Raisin pudding”;
Swiss cheese: vice versa

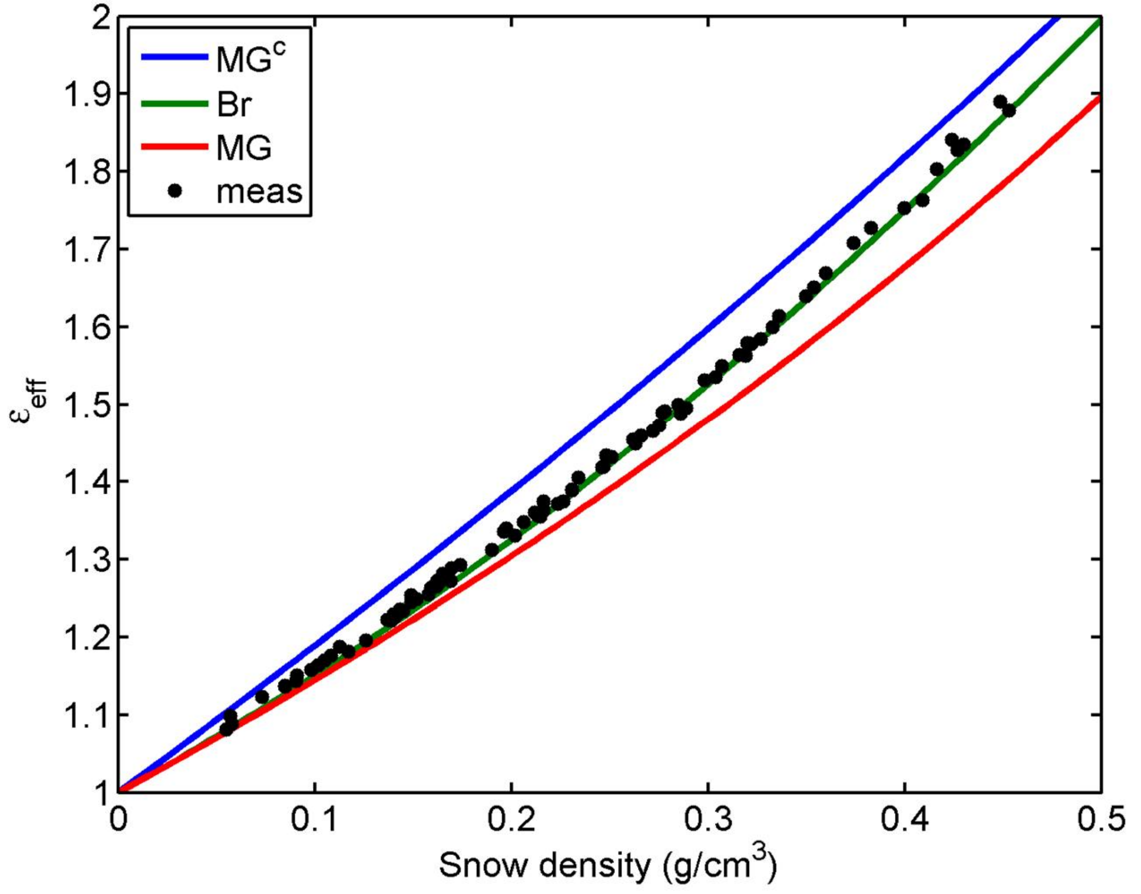
Raisin pudding : $\varepsilon_{\mathrm{i}} > \varepsilon_{\mathrm{e}}$

Z. Hashin and S. Shtrikman. A variational approach to the theory of the effective magnetic permeability of multiphase materials. *Journal of Applied Physics*, 33(10):3125–3131, 1962.

$$\varepsilon_{\mathrm{i}} = 20, \quad \varepsilon_{\mathrm{e}} = 1$$

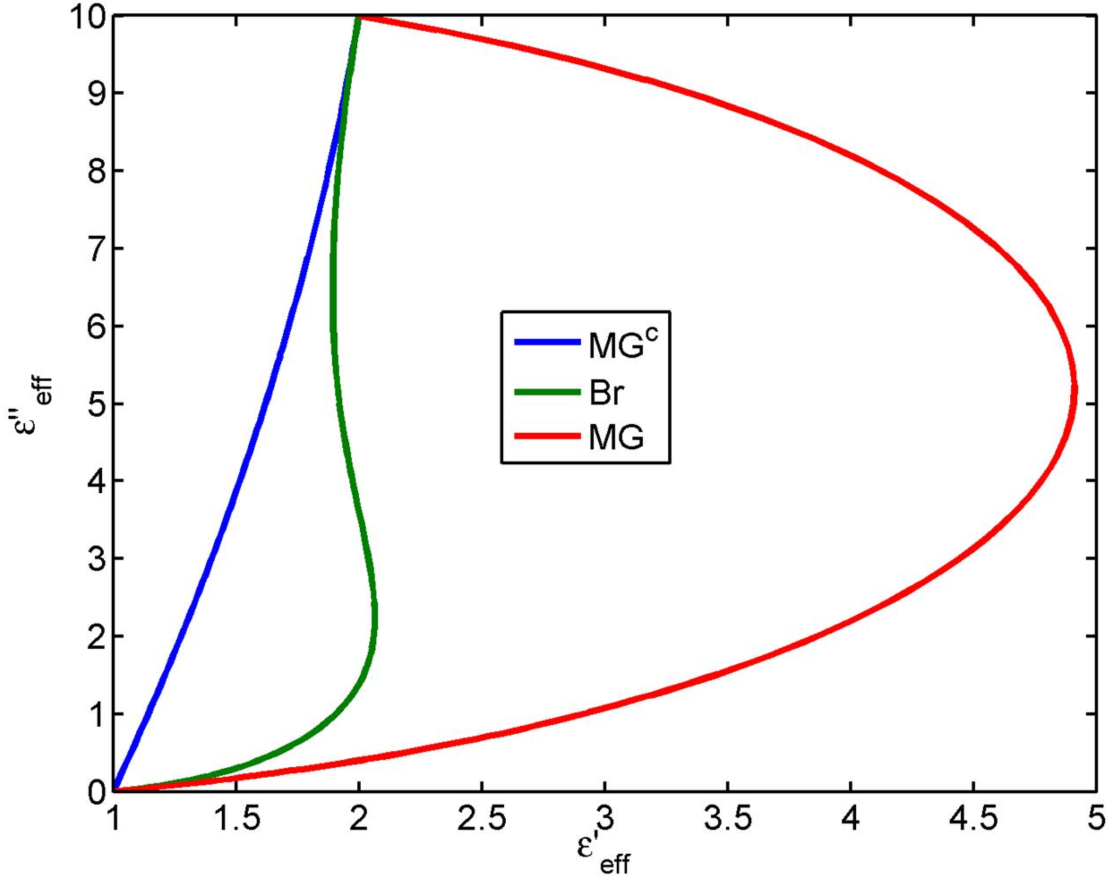


Example: dry snow

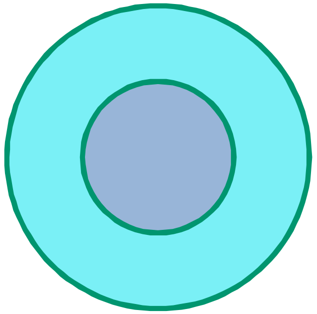


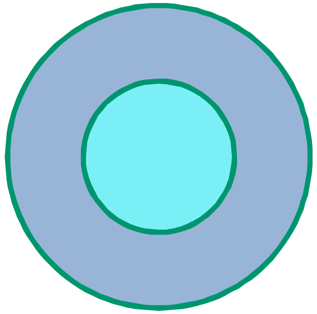
A. Sihvola, H. Wallén: Homogenization of amorphous media, in *Amorphous Nanophotonics* (Rockstuhl & Scharf, editors), Chapter 3, pp. 67–87, 2013.

Lossy mixtures



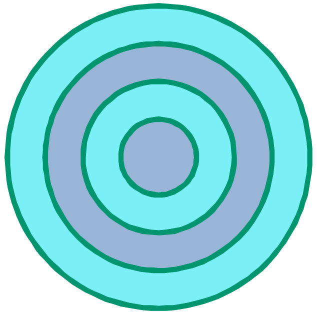
A. Sihvola, H. Wallén: Homogenization of amorphous media, in *Amorphous Nanophotonics* (Rockstuhl & Scharf, editors), Chapter 3, pp. 67–87, 2013.





$$\varepsilon = 2 \Rightarrow \alpha = \tfrac{3}{4} = 0.75$$

$$\varepsilon = 10 \Rightarrow \alpha = \tfrac{9}{4} = 2.25$$



50/50 - mixture:

$$\varepsilon_1 = 2, \varepsilon_2 = 10 \Rightarrow \alpha = \tfrac{51}{32} \approx 1.59$$

$$\varepsilon_1 = 10, \varepsilon_2 = 2 \Rightarrow \alpha = \tfrac{57}{32} \approx 1.78$$

$$\varepsilon_1 = 2, \varepsilon_2 = 10, \varepsilon_3 = 2 \Rightarrow \alpha = \tfrac{1587}{928} \approx 1.71$$

$$\varepsilon_1 = 10, \varepsilon_2 = 2, \varepsilon_3 = 10 \Rightarrow \alpha = \tfrac{909}{544} \approx 1.67$$

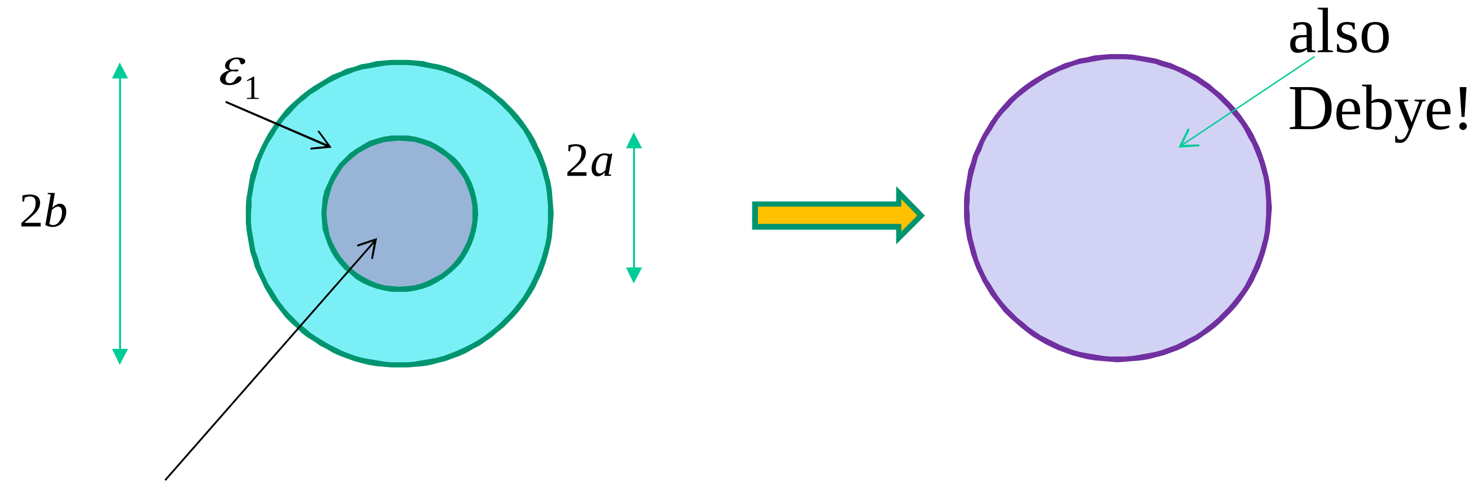
Polarizability design & limits

Repercussions:

- Bounds for the polarizability
- Dispersion engineering
- Cloaking applications

Mixing may affect dispersion strongly

- Drude spheres in insulating environment
  - Lorentz dispersion character
- Debye inclusions in dispersionless environment
  - mixture remains Debye, relaxation region shifts
- Maxwell–Wagner effect
  - interfacial polarization, emergent relaxation



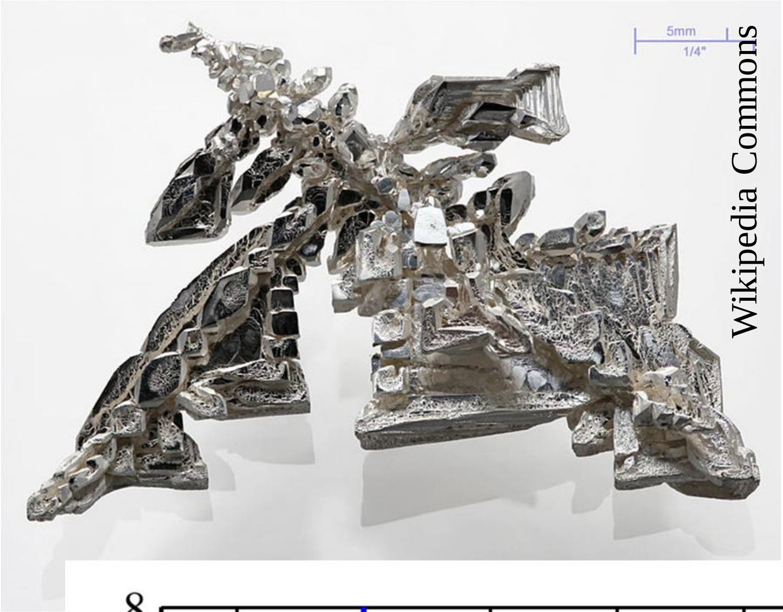
$$\varepsilon_D = \varepsilon_\infty + \frac{\varepsilon_S - \varepsilon_\infty}{1 + j\omega\tau}$$

$$\tau_c = \tau \frac{(\varepsilon_\infty + 2\varepsilon_1) b^2 - (\varepsilon_\infty - \varepsilon_1) a^2}{(\varepsilon_S + 2\varepsilon_1) b^2 - (\varepsilon_S - \varepsilon_1) a^2}$$

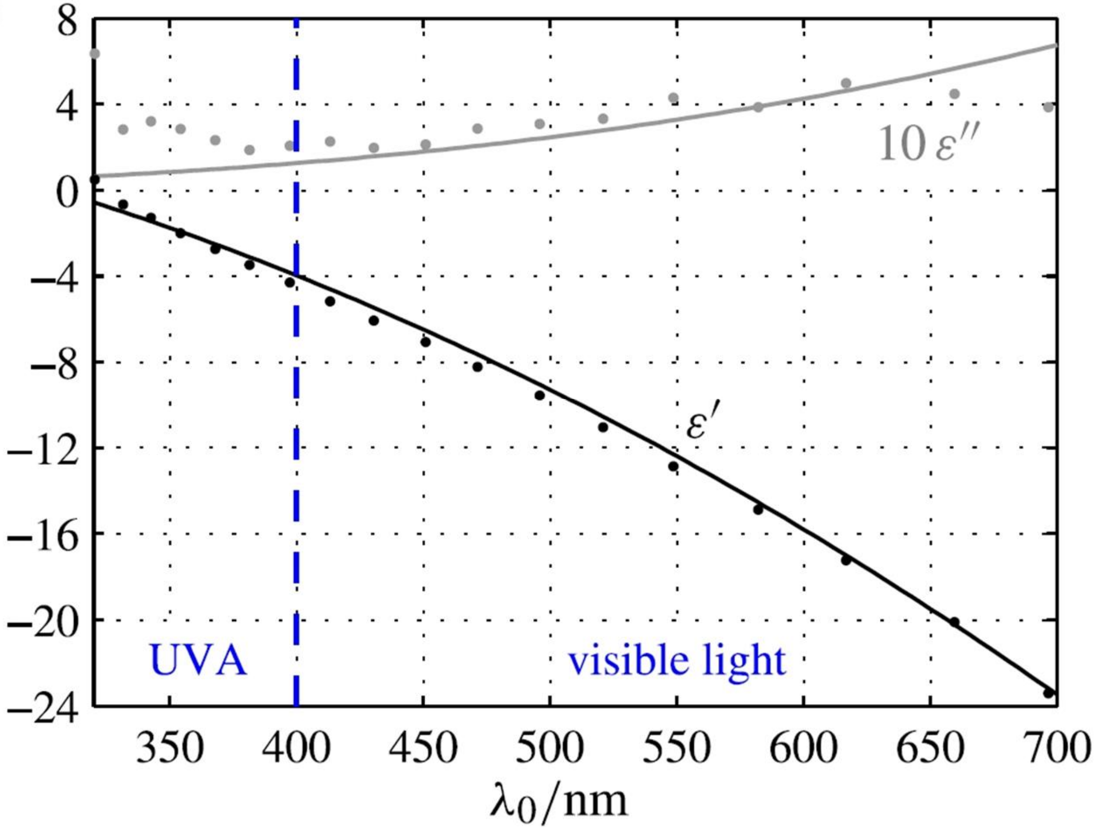
From mixing to single-particle response:

Modifying relaxation behavior

Silver

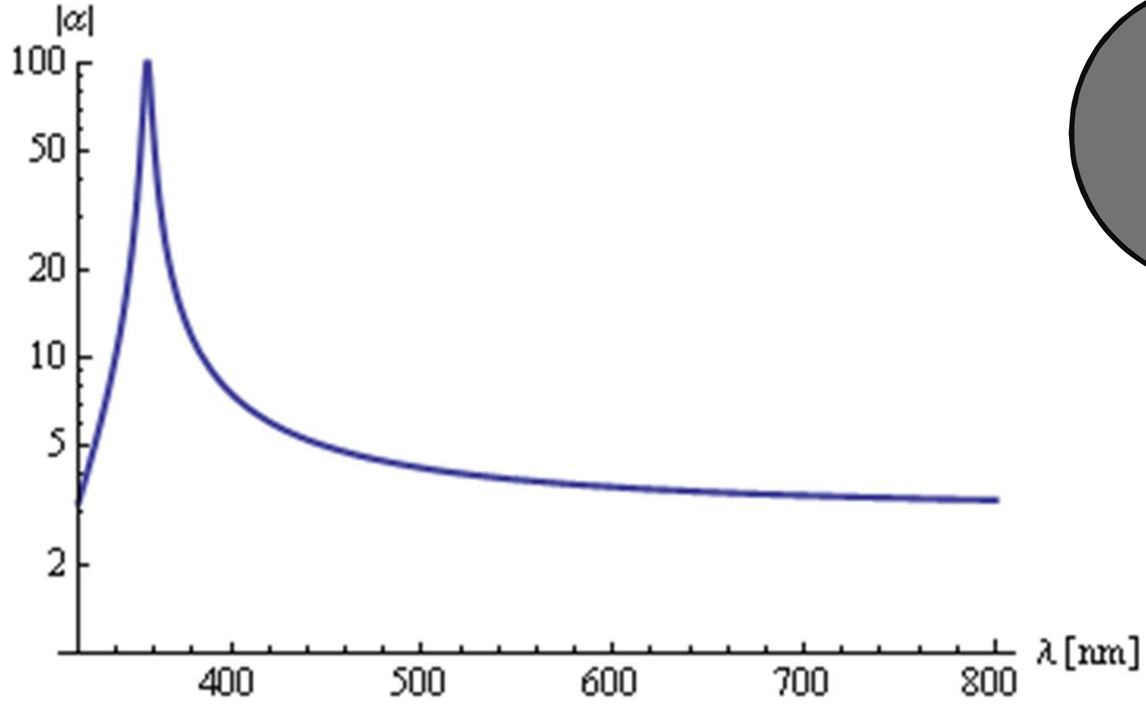


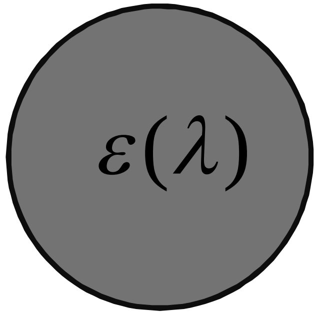
$$\varepsilon_r(\lambda) = \varepsilon_r' - j\,\varepsilon_r'' = \varepsilon_\infty - \frac{(\lambda/\lambda_p)^2}{1 - j\lambda/\lambda_d}\,, \qquad \begin{cases} \varepsilon_\infty = 5.5 \\ \lambda_p = 130\,\mathrm{nm} \\ \lambda_d = 30\,\mu\mathrm{m} \end{cases}$$



H. Wallén, H. Kettunen, A. Sihvola: Composite near-field superlens design using mixing formulas and simulations, *Metamaterials*, **3**(2009): 129-139. (modeled on the data by P.B. Johnson and R.W. Christy, *Physical Review B*, **6**(12), 4370-4379 (1972))

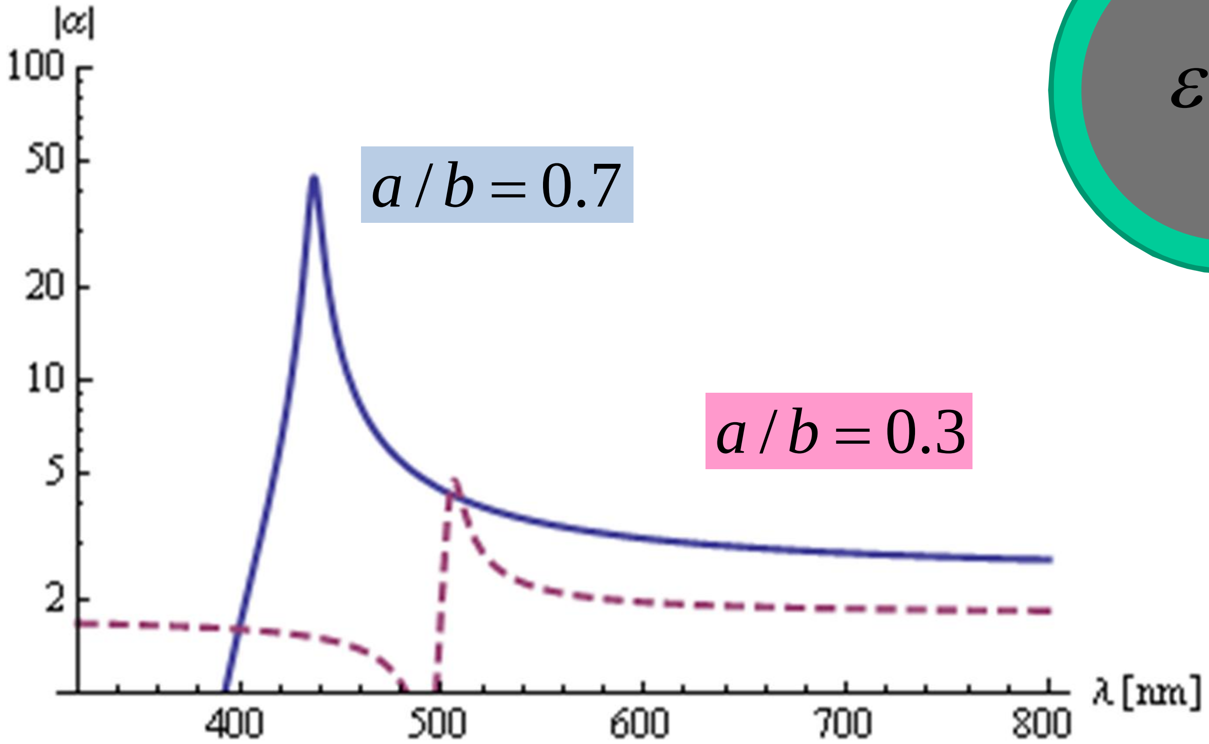
Redshifting silver glare...

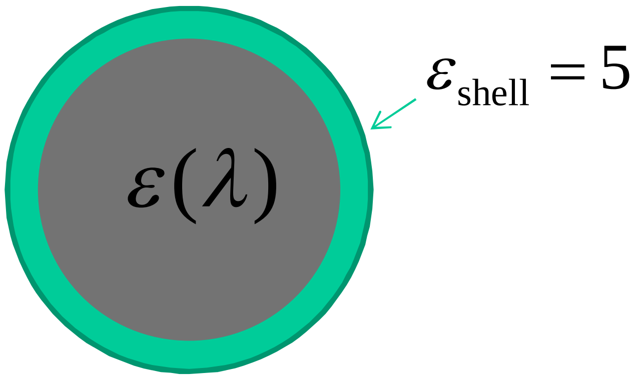


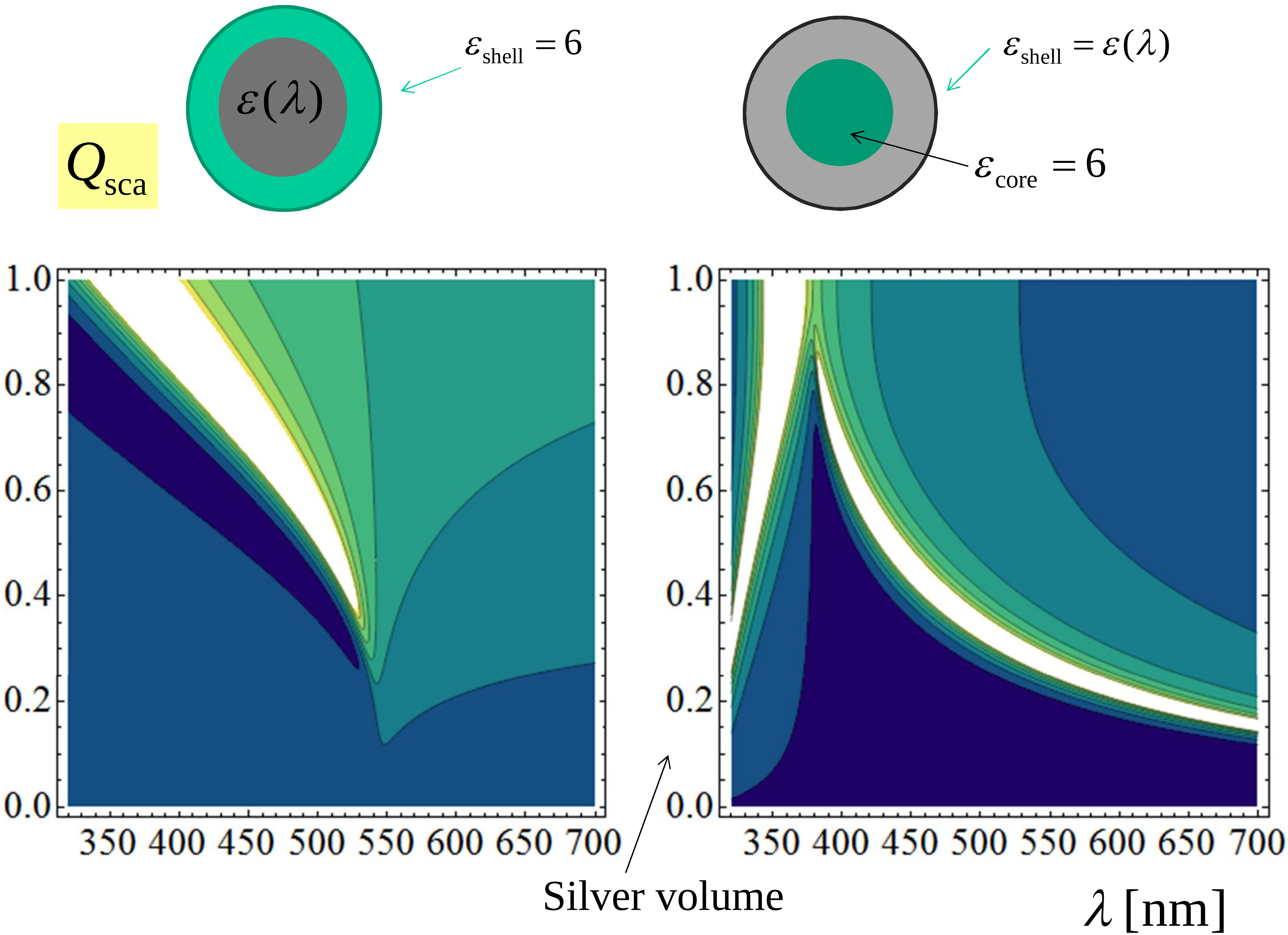


$$\alpha(\lambda) = 3\frac{\varepsilon(\lambda) - 1}{\varepsilon(\lambda) + 2}$$

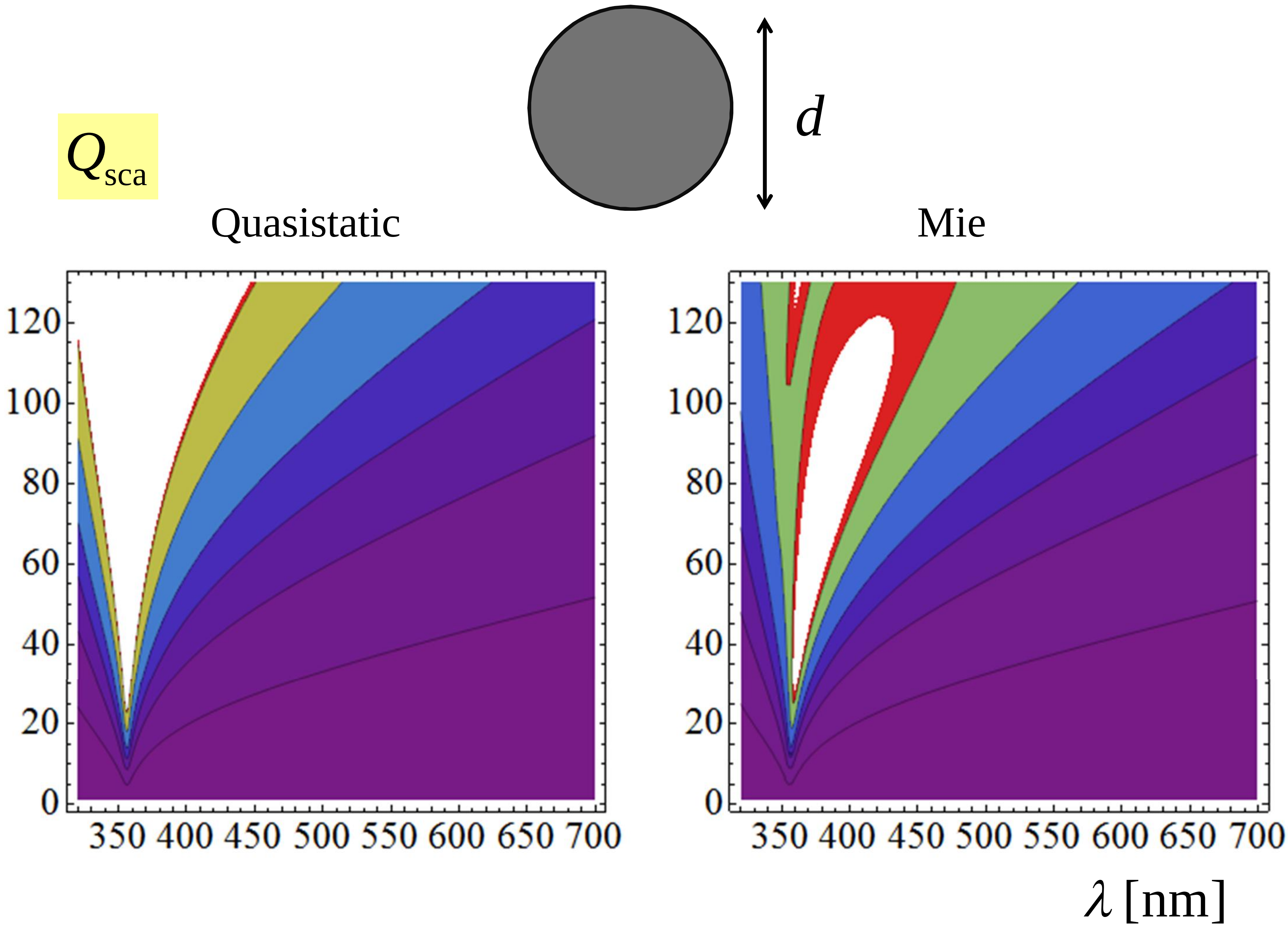
Redshifting silver glare...





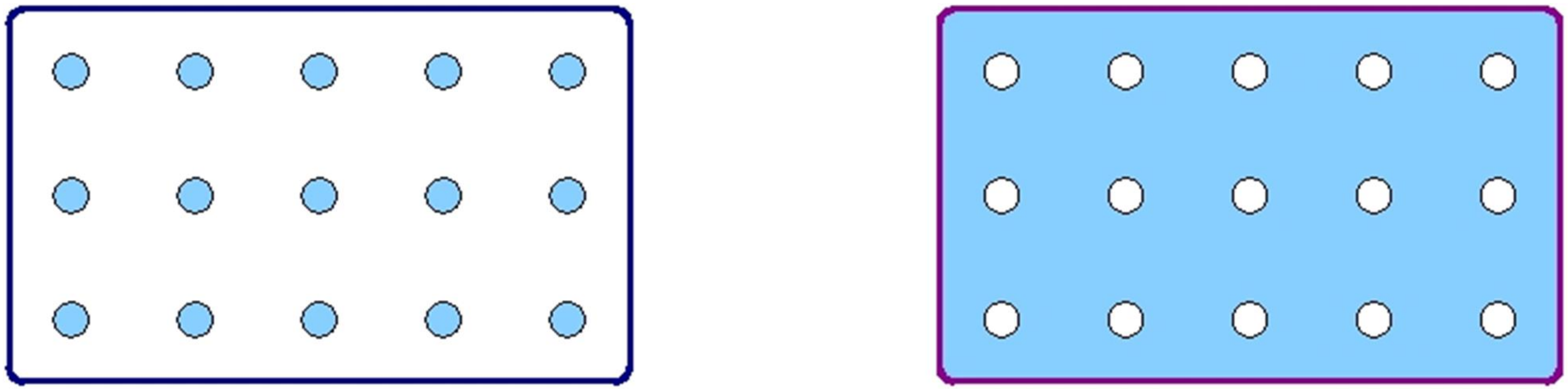


Q_{sca}
$\varepsilon(\lambda)$
$\varepsilon_{shell} = 6$
$\varepsilon_{shell} = \varepsilon(\lambda)$
$\varepsilon_{core} = 6$
1.0
0.8
0.6
0.4
0.2
0.0
350 400 450 500 550 600 650 700
Silver volume
λ [nm]



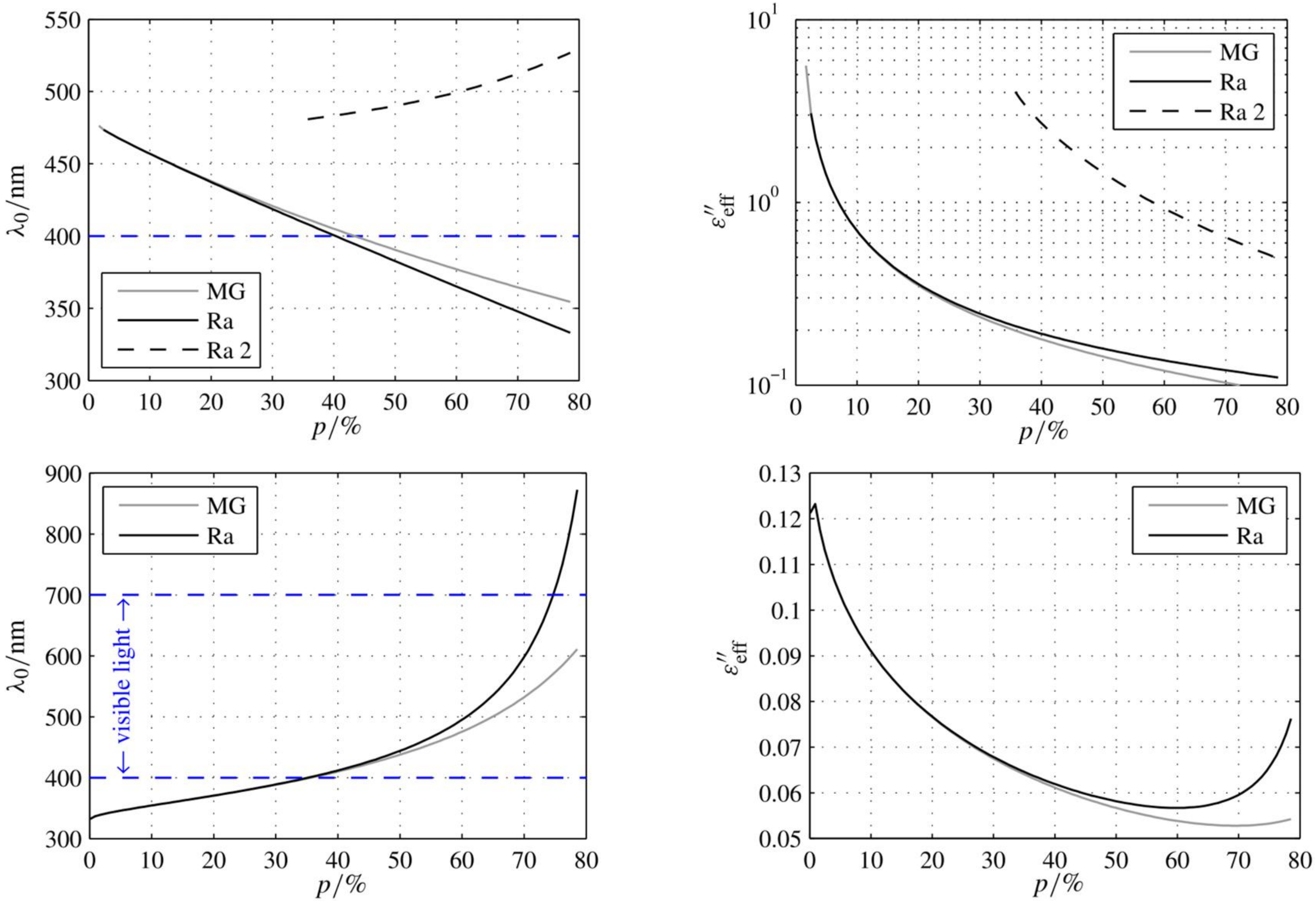
Q_{sca}
d
Quasistatic
Mie
0
20
40
60
80
100
120
350
400
450
500
550
600
650
700
λ [nm]

Modification of plasma parameters in negative-permittivity composites



H. Wallén, H. Kettunen, A. Sihvola: Composite near-field superlens design using mixing formulas and simulations, *Metamaterials*, **3**(2009): 129-139.

Tuning of plasma wavelength: $\varepsilon' = -1$



Raisin pudding

Swiss cheese

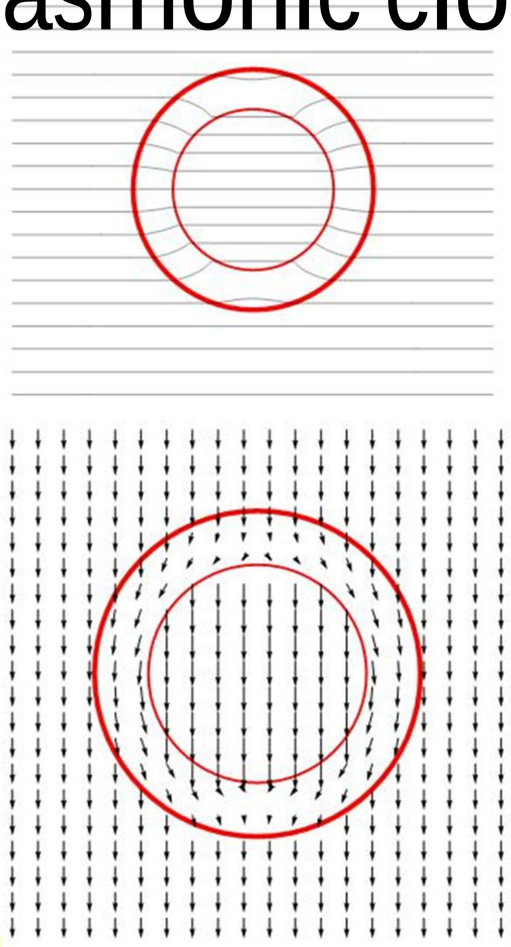
H. Wallén, H. Kettunen, A. Sihvola: Composite near-field superlens design using mixing formulas and simulations, *Metamaterials*, **3**(2009): 129-139.

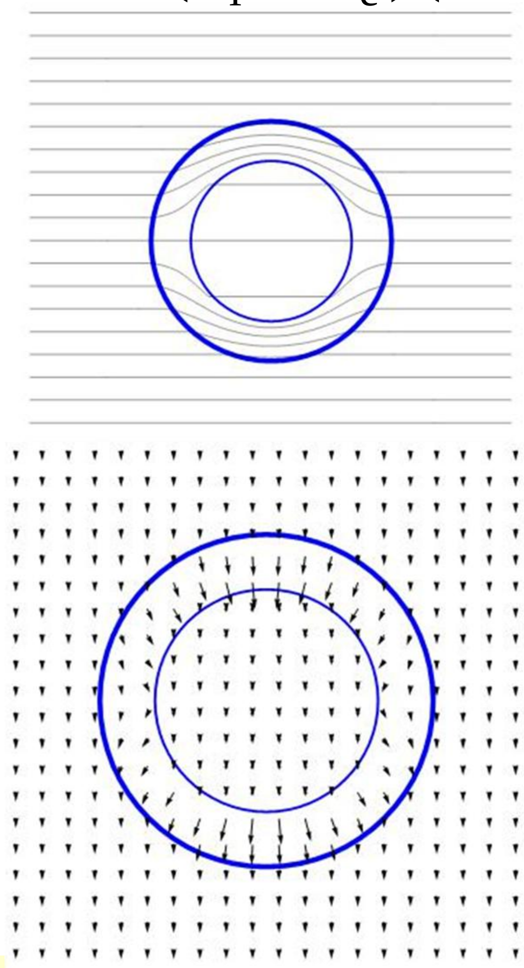
Repercussions:

- Bounds for the polarizability
- Dispersion engineering
- Cloaking applications

Invisible inclusions and plasmonic cloaking

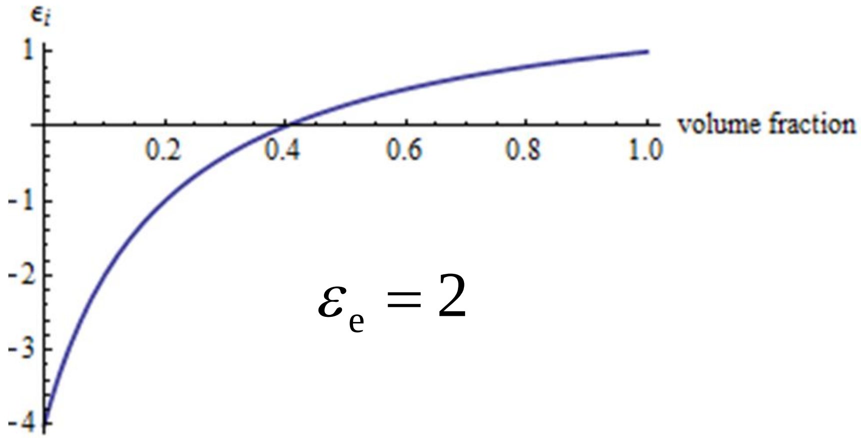
$$p = \frac{(\varepsilon_\mathrm{i} - 2\varepsilon_\mathrm{e})(1 - \varepsilon_\mathrm{e})}{(\varepsilon_\mathrm{i} - \varepsilon_\mathrm{e})(1 + 2\varepsilon_\mathrm{e})}$$

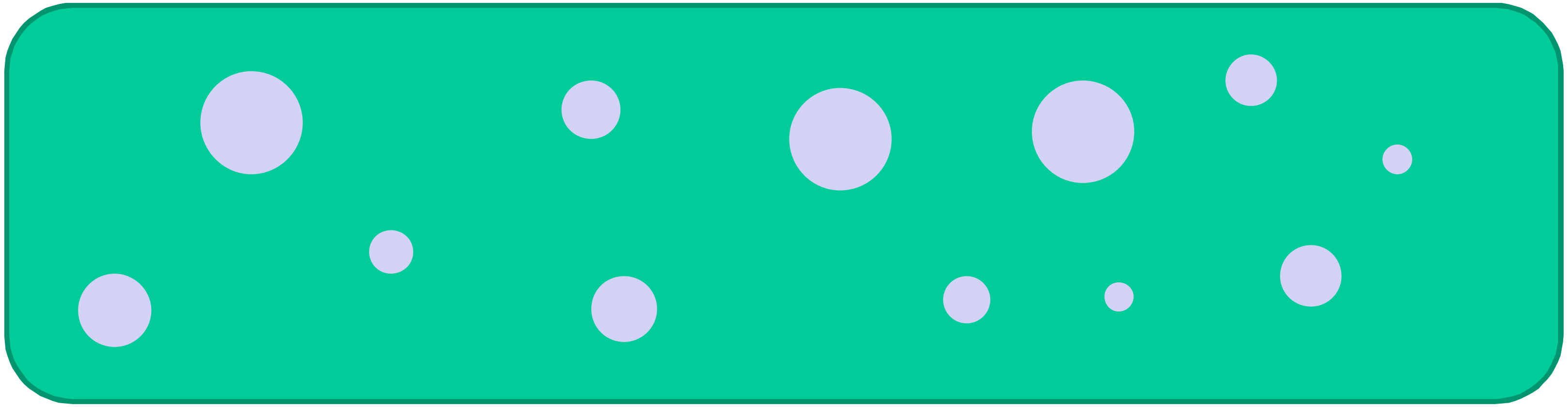




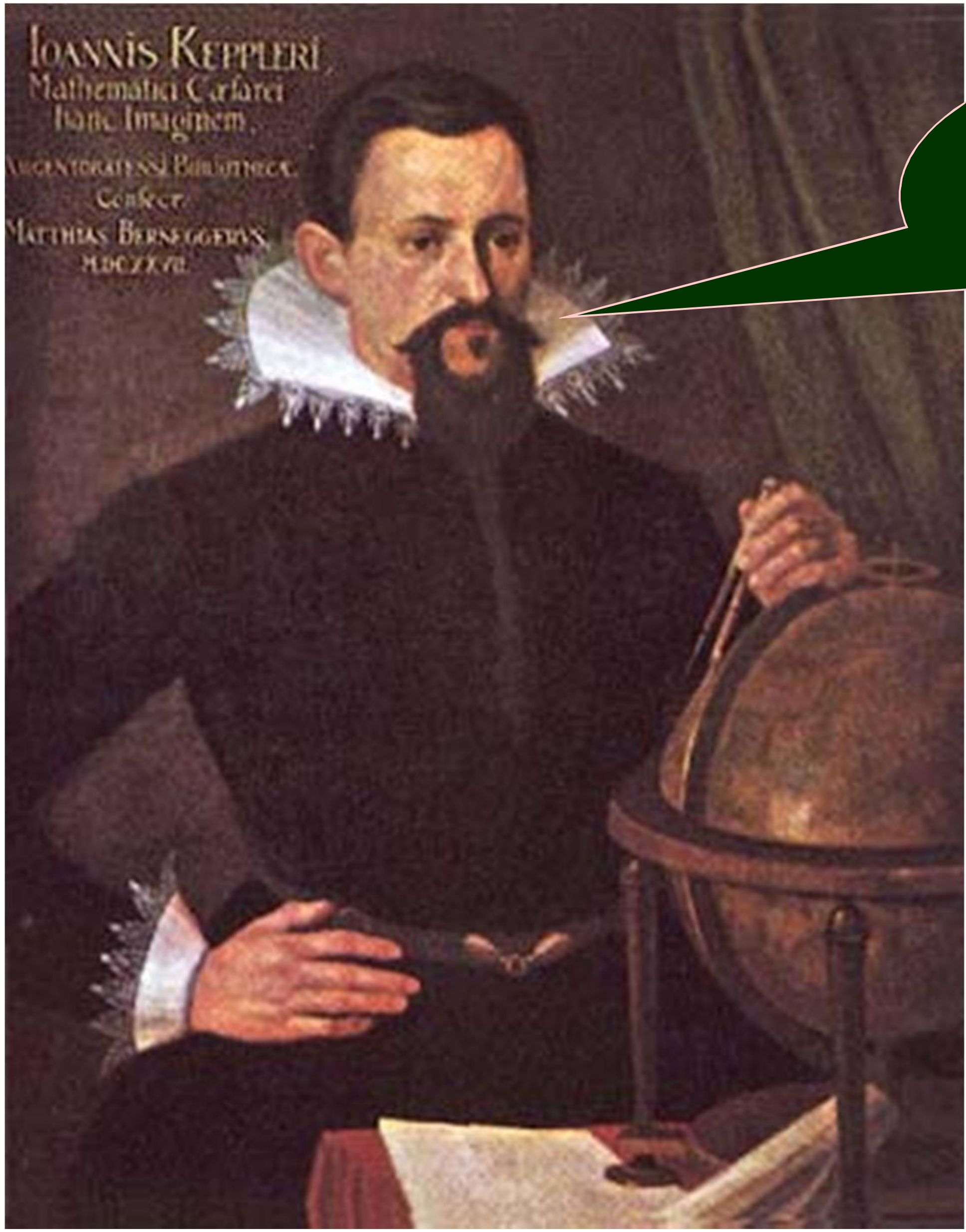
A. Sihvola: Properties of dielectric mixtures with layered spherical inclusions, in *Microwave radiometry and remote sensing applications, (*P. Pampaloni, editor), Utrecht: VSP, 1989, pp. 115–123.
A. Alù and N. Engheta: Plasmonic and metamaterial cloaking: physical mechanisms and potentials," *Journal of Optics A: Pure and Applied Optics*, 10(9)093002, 2008.

$$p = \frac{(\varepsilon_{\mathrm{i}} - 2\varepsilon_{\mathrm{e}})(1 - \varepsilon_{\mathrm{e}})}{(\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}})(1 + 2\varepsilon_{\mathrm{e}})}$$





Transparent composites with the same recipé



Ubi materia, ibi geometria

"where there is matter, there is geometry"

Johannes Kepler
(1571–1630)

A. Sihvola: Ubi materia, ibi geometria, http://users.aalto.fi/asihvola/umig.pdf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Espoo, Finland
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